MATH 226: Notes on Assignment 3

Practice Problems 1.3: 1-31 odd

1. The differential equation is second order, since the highest derivative in the equation is of order two. The equation is linear since the left hand side is a linear function of y and its derivatives and the right hand side is just a function of t.

3. The differential equation is fourth order since the highest derivative in the equation is of order four. The equation is linear since the left hand side is a linear function of y and its derivatives and the right hand side

5. The differential equation is second order since the highest derivative in the equation is of order two. The equation is nonlinear because of the term $\sin(t + y)$ which is not a linear function of y.

7. $a_0 = 1$, $a_1 = 1/(1+t)$, $g = 2 \sin t$; nonhomogeneous.

9. $a_0 = x^2$, $a_1 = -3x$, $a_2 = 4$, $g = \ln x$; nonhomogeneous.

11. $a_0 = 1, a_1 = 0, a_2 = \cos t$, $a_3 = 0$, $a_4 = 1$, $g = e^{-t} \sin t$; nonhomogenous.

13. If $y_1 = e^t$, then $y_1' = e^t$ and $y_1'' = e^t$. Therefore, $y_1'' - y_1 = 0$. Also, $y_2 = \cosh t$ implies $y_2' = \sinh t$ and $y_2'' = \cosh t$. Therefore, $y_2'' - y_2 = 0$.

15. If $y = 3t + t^2$, then $y' = 3+2t$. Thus, $ty' - y = t(3 + 2t) - (3t + t^2) = t^2$.

17. If $y = t^{1/2}$, then $y' = t^{-1/2}/2$ and $y'' = -t^{-3/2}/4$. Therefore,
$2t^2y'' + 3ty' - y = 2t^2(-t^{-3/2}/4) + 3t(t^{-1/2}/2) - t^{1/2} = (-1/2+3/2-1)t^{1/2} = 0$.
Also, $y_2 = t^{-1}$ implies $y_2' = -t^{-2}$ and $y'' = 2t^{-3}$. Therefore, $2t^2y'' + 3ty' - y = 2t^2(2t^{-3}) + 3t(-t^{-2}) - t^{-1} = (4 - 3 - 1)t^{-1} = 0$.

19. If $y = (\cos t) \ln \cos t + t \sin t$, then $y' = -(\sin t) \ln \cos t + (\cos t)(-\sin t)/(\cos t) + 1 \sin t + t \cos t$
$= -(\sin t) \ln \cos t - \sin t + \sin t + t \cos t = -(\sin t) \ln \cos t + t \cos t$ and
$y'' = -(\cos t) \ln \cos t + (-\sin t)(-\sin t)/\cos t + 1(\cos t) + t(-\sin t)$.
Thus, $y'' + y = -(\cos t) \ln \cos t - t \sin t + \sin^2 t/\cos t + \cos t + (\cos t) \ln \cos t + t \sin t$

$$= \frac{\sin^2 t}{\cos t} + \cos t = \frac{\sin^2 t + \cos^2 t}{\cos t} = \frac{1}{\cos t} = \sec t$$

21. Let $y = e^{rt}$. Then $y' = re^{rt}$. Substituting these terms into the differential equation, we have $y' + 2y = re^{rt} + 2e^{rt} = (r+2)e^{rt} = 0$. This equation implies $r = -2$ since e^{rt} is always positive.

23. Let $y = e^{rt}$. Then $y' = r\,e^{rt}$ and $y'' = r^2\,e^{rt}$. Substituting these terms into the differential equation, we have $y'' + y' - 6y = (r^2 + r - 6)e^{rt} = 0$. In order for r to satisfy this equation, we need $r^2 + r - 6 = (r-2)(r+3) = 0$. That is, we need $r = 2, -3$.

25. Let $y = t^r$. Then $y' = rt^{r-1}$ and $y'' = r(r-1)t^{r-2}$. Substituting these terms into the differential equation, we have $t^2y'' + 4ty' + 2y = t^2(r(r-1)t^{r-2}) + 4t(rt^{r-1}) + 2t^r = (r(r-1)+4r+2)t^r = 0$. In order for r to satisfy this equation, we need $r(r-1)+4r+2 = 0$. Simplifying this expression, we need $r^2 + 3r + 2 = 0$. The solutions of this equation are $r = -1, -2$.

27. If $y = Ce^{-2t}$, then $y' = -2Ce^{-2t}$. Thus $y' + 2y = 0$. Also, $1 = y(0) = Ce^0 = C$.

29. If $y = (\sin t)/t^2 + C/t^2$, then $y' = (\cos t)/t^2 - 2(\sin t)/t^3 - 2C/t^3$. Thus $y' + (2/t)y = (\cos t)/t^2$. Also, $1/2 = y(1) = (\sin 1)/1 + C/1$ so $C = ½ - \sin 1$.

31. If $y = e^{-t^2/4}\int_0^t e^{s^2/4}\,ds + Ce^{-t^2/4}$, then $y' = e^{-t^2/4}(-2t/4)\int_0^t e^{s^2/4}\,ds + e^{-t^2/4}e^{t^2/4} + Ce^{-t^2/4}(-2t/4) = e^{-t^2/4}\left(\frac{-t}{2}\right)\int_0^t e^{s^2/4}\,ds + 1 + Ce^{-t^2/4}\left(\frac{-t}{2}\right)$ and $2y' = -te^{-t^2/4}\int_0^t e^{s^2/4}\,ds + +2 - tCe^{-t^2/4}$ and $ty = te^{-t^2/4}\int_0^t e^{s^2/4}\,ds + tCe^{-t^2/4}$ so $2y' + ty = 2$. Also $1 = y(0) = e^0\int_0^o e^{s^2/4}\,ds + Ce^0 = 1\bullet 0 + C$ so $C = 1$