MATH 225: *Notes on Assignment 18*

7.2 Almost Linear Systems

Practice Problems: 1*, 3*, 6*, 10*, 12*, 13*, 16*, 25, 26

Feedback Problems: 12*, 16*, 25

1.(a) The equation $dx/dt = 0$ implies $y = 2x$. The equation $dy/dt = 0$ implies $y = x^2$. Therefore, for these equations to both be satisfied, we need $2x = x^2$ which means $x = 0$ or $x = 2$. Thus the two critical points are $(0,0)$ and $(2,4)$.

(b) Here, we have $F(x,y) = -2x + y$ and $G(x,y) = x^2 - y$. Therefore, the Jacobian matrix for this system is

$$\mathbf{J}(x,y) = \begin{pmatrix} F_x & F_y \\ G_x & G_y \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 2x & -1 \end{pmatrix}.$$

Near the critical point $(0,0)$, the Jacobian matrix is

$$\mathbf{J}(0,0) = \begin{pmatrix} F_x(0,0) & F_y(0,0) \\ G_x(0,0) & G_y(0,0) \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 0 & -1 \end{pmatrix}$$

and the corresponding linear system near $(0,0)$ is

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

Near the critical point $(2,4)$, the Jacobian matrix is

$$\mathbf{J}(2,4) = \begin{pmatrix} F_x(2,4) & F_y(2,4) \\ G_x(2,4) & G_y(2,4) \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 4 & -1 \end{pmatrix}$$

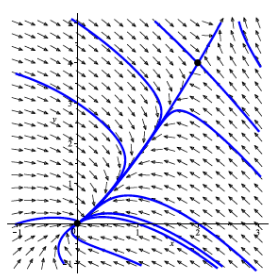
and the corresponding linear system near $(2,4)$ is

$$\frac{d}{dt}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$$

where $u = x - 2$ and $v = y - 4$.

(c) The eigenvalues of the linear system near $(0,0)$ are $\lambda = -1, -2$. From this, we can conclude that $(0,0)$ is an asymptotically stable node for the nonlinear system. The eigenvalues of the linear system near $(2,4)$ are $(-3 \pm \sqrt{17})/2$. Since one of these eigenvalues is positive and one is negative, the critical point $(2,4)$ is an unstable saddle point for the nonlinear system.

(d)



(e) The basin of attraction for the asymptotically stable critical point $(0,0)$ is bounded on the right by trajectories heading towards the critical point $(2,4)$.

3.(a) To find the critical points, we need to solve the equations $x = -y^2$ and $x = -2y$. In order for these two equations to be satisfied simultaneously, we need $y^2 = 2y$. Therefore, $y = 0$ or $y = 2$. Therefore, the two critical points are $(0,0)$ and $(-4,2)$.

(b) Here, we have $F(x,y) = x + y^2$ and $G(x,y) = x + 2y$. Therefore, the Jacobian matrix for this system is

$$\mathbf{J}(x,y) = \begin{pmatrix} F_x & F_y \\ G_x & G_y \end{pmatrix} = \begin{pmatrix} 1 & 2y \\ 1 & 2 \end{pmatrix}.$$

Near the critical point $(0,0)$, the Jacobian matrix is

$$\mathbf{J}(0,0) = \begin{pmatrix} F_x(0,0) & F_y(0,0) \\ G_x(0,0) & G_y(0,0) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$$

and the corresponding linear system near $(0,0)$ is

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

Near the critical point $(-4,2)$, the Jacobian matrix is

$$\mathbf{J}(-4,2) = \begin{pmatrix} F_x(-4,2) & F_y(-4,2) \\ G_x(-4,2) & G_y(-4,2) \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 1 & 2 \end{pmatrix}$$

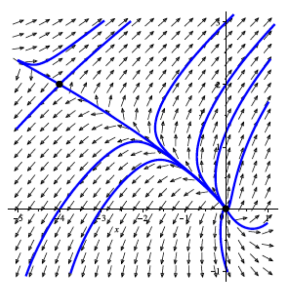
and the corresponding linear system near $(-4,2)$ is

$$\frac{d}{dt}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$$

where $u = x + 4$ and $v = y - 2$.

(c) The eigenvalues of the linear system near $(0,0)$ are $\lambda = 1, 2$. From this, we can conclude that $(0,0)$ is an unstable node for the nonlinear system. The eigenvalues of the linear system near $(-4,2)$ are $\lambda = (3 \pm \sqrt{17})/2$. Since one of these eigenvalues is positive and one is negative, the critical point $(-4,2)$ is an unstable saddle point for the nonlinear system.

(d)



6.(a) To find the critical points, we need to solve the equations $x - x^2 - xy = 0$ and $3y - xy - 2y^2 = 0$. Solving this system of equations, we see that the critical points are given by $(0,0)$, $(0,3/2)$, $(1,0)$, and $(-1,2)$.

(b) Here, we have $F(x,y) = x - x^2 - xy$ and $G(x,y) = 3y - xy - 2y^2$. Therefore, the Jacobian matrix for this system is

$$\mathbf{J}(x,y) = \begin{pmatrix} F_x & F_y \\ G_x & G_y \end{pmatrix} = \begin{pmatrix} 1-2x-y & -x \\ -y & 3-x-4y \end{pmatrix}.$$

Near the critical point $(0,0)$, the Jacobian matrix is

$$\mathbf{J}(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

and the corresponding linear system near $(0,0)$ is

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

Near the critical point $(0,3/2)$, the Jacobian matrix is

$$\mathbf{J}(0,3/2) = \begin{pmatrix} -1/2 & 0 \\ -3/2 & -3 \end{pmatrix}$$

and the corresponding linear system near $(0,3/2)$ is

$$\frac{d}{dt}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -1/2 & 0 \\ -3/2 & -3 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$$

where $u = x$ and $v = y - 3/2$. Near the critical point $(1,0)$, the Jacobian matrix is

$$\mathbf{J}(1,0) = \begin{pmatrix} -1 & -1 \\ 0 & 2 \end{pmatrix}$$

and the corresponding linear system near $(1,0)$ is

$$\frac{d}{dt}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$$

where $u = x - 1$ and $v = y$. Near the critical point $(-1,2)$, the Jacobian matrix is

$$\mathbf{J}(-1,2) = \begin{pmatrix} 1 & 1 \\ -2 & -4 \end{pmatrix}$$

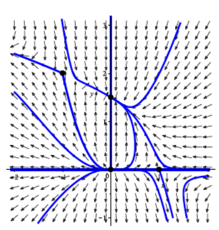
and the corresponding linear system near $(-1,2)$ is

$$\frac{d}{dt}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -2 & -4 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$$

where $u = x + 1$ and $v = y - 2$.

(c) The eigenvalues of the linear system near $(0,0)$ are $\lambda = 1, 3$. From this, we can conclude that $(0,0)$ is an unstable node for the nonlinear system. The eigenvalues of the linear system near $(0,3/2)$ are $\lambda = -1/2, -3$. From this, we can conclude that $(0,3/2)$ is an asymptotically stable node for the nonlinear system. The eigenvalues of the linear system near $(1,0)$ are $\lambda = -1, 2$. From this, we can conclude that $(1,0)$ is a saddle point for the nonlinear system. The eigenvalues of the linear system near $(-1,2)$ are $\lambda = (-3 \pm \sqrt{17})/2$. From this, we can conclude that $(-1,2)$ is a saddle point for the nonlinear system.

(d)



(e) The basin of attraction for the asymptotically stable point $(0,3/2)$ consists of the first quadrant combined with trajectories heading into the second quadrant from $(0,0)$ and towards $(0,3/2)$.

10.(a) To find the critical points, we need to solve the equations $x+x^2+y^2=0$ and $y-xy=0$. Solving these equations, we find that the critical points are $(0,0)$ and $(-1,0)$.

(b) Here, we have $F(x,y)=x+x^2+y^2$ and $G(x,y)=y-xy$. Therefore, the Jacobian matrix for this system is

$$\mathbf{J}(x,y)=\begin{pmatrix} F_x & F_y \\ G_x & G_y \end{pmatrix}=\begin{pmatrix} 1+2x & 2y \\ -y & 1-x \end{pmatrix}.$$

Near the critical point $(0,0)$, the Jacobian matrix is

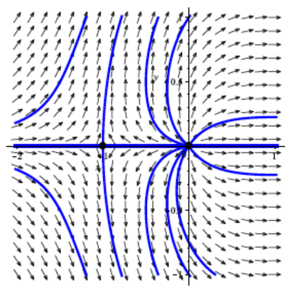
$$\mathbf{J}(0,0)=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Near the critical point $(-1,0)$, the Jacobian matrix is

$$\mathbf{J}(-1,0)=\begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix}.$$

(c) The eigenvalues of the linear system near $(0,0)$ are $\lambda=1$. From this, we can conclude that $(0,0)$ is an unstable node or spiral point for the nonlinear system. The eigenvalues of the linear system near $(-1,0)$ are $\lambda=-1,3$. From this, we can conclude that $(-1,0)$ is a saddle point for the nonlinear system.

(d)



12.(a) To find the critical points, we need to solve the equations $(2+x)\sin y=0$ and $1-x-\cos y=0$. If $x=-2$, then we must have $\cos y=3$, which is impossible. Therefore, $\sin y=0$, which implies that $y=n\pi$, $n=0,\pm1,\pm2,\ldots$ Based on the second equation, $x=1-\cos n\pi$. It follows that the critical points are located at $(0,2k\pi)$ and $(2,(2k+1)\pi)$ where $k=0,\pm1,\pm2,\ldots$

(b) Here, we have $F(x,y)=(2+x)\sin y$ and $G(x,y)=1-x-\cos y$. Therefore, the Jacobian matrix for this system is

$$\mathbf{J}(x,y)=\begin{pmatrix} F_x & F_y \\ G_x & G_y \end{pmatrix}=\begin{pmatrix} \sin y & (2+x)\cos y \\ -1 & \sin y \end{pmatrix}.$$

Near the critical points $(0,2k\pi)$, the Jacobian matrix is

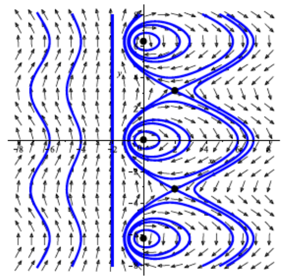
$$\mathbf{J}(0,2k\pi)=\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}.$$

Near the critical point $(2,(2k+1)\pi)$, the Jacobian matrix is

$$\mathbf{J}(2,(2k+1)\pi)=\begin{pmatrix} 0 & -4 \\ -1 & 0 \end{pmatrix}.$$

(c) The eigenvalues of the linear system near $(0,2k\pi)$ are $\lambda=\pm\sqrt{2}i$. Based on this information, we cannot make a conclusion about the nature of the critical points near $(0,2k\pi)$ for the nonlinear system. The eigenvalues of the linear system near $(2,(2k+1)\pi)$ are $\lambda=\pm2$. From this, we can conclude that the critical points $(2,(2k+1)\pi)$ are saddles.

(d)



Upon looking at the phase portrait, we see that the critical points $(0, 2k\pi)$ are centers.

13.(a) To find the critical points, we need to solve the equations $x - y^2 = 0$ and $y - x^2 = 0$. Substituting $y = x^2$ into the first equation, results in $x - x^4 = 0$ which has real roots $x = 0, 1$. Therefore, the critical points are $(0, 0)$ and $(1, 1)$.

(b) Here, we have $F(x, y) = x - y^2$ and $G(x, y) = y - x^2$. Therefore, the Jacobian matrix for this system is

$$\mathbf{J}(x, y) = \begin{pmatrix} F_x & F_y \\ G_x & G_y \end{pmatrix} = \begin{pmatrix} 1 & -2y \\ -2x & 1 \end{pmatrix}.$$

Near the critical point $(0, 0)$, the Jacobian matrix is

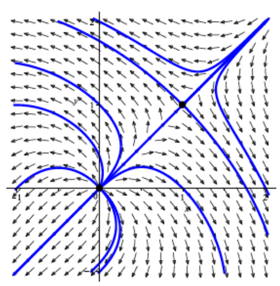
$$\mathbf{J}(0, 0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Near the critical point $(1, 1)$, the Jacobian matrix is

$$\mathbf{J}(1, 1) = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}.$$

(c) There is a repeated eigenvalue $\lambda = 1$ for the linear system near $(0, 0)$. Based on this information, we cannot make a conclusion about the nature of the critical point near $(0, 0)$ for the nonlinear system, only that it is unstable and it is either a node or a spiral. The eigenvalues of the linear system near $(1, 1)$ are $\lambda = 3, -1$. From this, we can conclude that the critical point $(1, 1)$ is a saddle.

(d)



Upon looking at the phase portrait, we see that the critical point $(0, 0)$ is an unstable node.

16.(a) To find the critical points, we need to solve the equations

$$\begin{aligned} y + x(1 - x^2 - y^2) &= 0 \\ -x + y(1 - x^2 - y^2) &= 0. \end{aligned}$$

Multiplying the first equation by y, the second equation by x and subtracting the second from the first, we conclude that $x^2 + y^2 = 0$. Therefore, the only critical point is $(0, 0)$.

(b) Here, we have $F(x,y) = y + x(1-x^2-y^2)$ and $G(x,y) = -x + y(1-x^2-y^2)$. Therefore, the Jacobian matrix for this system is

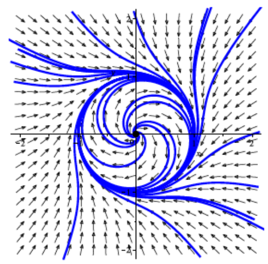
$$\mathbf{J}(x,y) = \begin{pmatrix} F_x & F_y \\ G_x & G_y \end{pmatrix} = \begin{pmatrix} 1-3x^2-y^2 & 1-2xy \\ -1-2xy & 1-x^2-3y^2 \end{pmatrix}.$$

Near the critical point $(0,0)$, the Jacobian matrix is

$$\mathbf{J}(0,0) = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

(c) The eigenvalues of the linear system near $(0,0)$ are $\lambda = 1 \pm i$. Therefore, $(0,0)$ is an unstable spiral point for the nonlinear system.

(d)



25. The characteristic equation for the coefficient matrix is $\lambda^2 + 1 = 0$, which has roots $\lambda = \pm i$. Therefore, the critical point at the origin is a center. For the perturbed matrix, the characteristic equation is $\lambda^2 - 2\epsilon\lambda + 1 + \epsilon^2 = 0$. This equation has roots $\lambda = \epsilon \pm i$. Therefore, as long as $\epsilon \neq 0$, the critical point of the perturbed system is a spiral point. Its stability depends on the sign of ϵ.

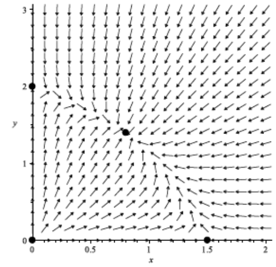
26. The characteristic equation for the coefficient matrix is $(\lambda + 1)^2 = 0$, which has the repeated root $\lambda = -1$. Therefore, the critical point is an asymptotically stable node. For the perturbed matrix, the characteristic equation is $\lambda^2 + 2\lambda + 1 + \epsilon = 0$. This equation has roots $\lambda = -1 \pm \sqrt{-\epsilon}$. If $\epsilon > 0$, then the roots are complex and the critical point is a stable spiral. If $\epsilon < 0$, then the roots are real and both negative, in which case the critical point remains a stable node.

7.3 Competing Species

Practice Problems: 1, 3, 5, 10

Feedback Problems: 1, 3, 5, 10

1.(a)



(b) The critical points are solutions of the system

$$\begin{aligned} x(1.5 - x - 0.5y) &= 0 \\ y(2 - y - 0.75x) &= 0. \end{aligned}$$

The four critical points are $(0,0)$, $(0,2)$, $(1.5,0)$, and $(0.8,1.4)$.

(c) The Jacobian matrix is

$$\mathbf{J}(x,y) = \begin{pmatrix} 3/2 - 2x - y/2 & -x/2 \\ -3y/4 & 2 - 3x/4 - 2y \end{pmatrix}.$$

At $(0,0)$,

$$\mathbf{J}(0,0) = \begin{pmatrix} 3/2 & 0 \\ 0 & 2 \end{pmatrix}.$$

The associated eigenvalues and eigenvectors are $\lambda_1 = 3/2$, $\mathbf{v}_1 = (1,0)^T$ and $\lambda_2 = 2$, $\mathbf{v}_2 = (0,1)^T$. The eigenvalues are positive. Therefore, the origin is an unstable node. At $(0,2)$,

$$\mathbf{J}(0,2) = \begin{pmatrix} 1/2 & 0 \\ -3/2 & -2 \end{pmatrix}.$$

The associated eigenvalues and eigenvectors are $\lambda_1 = 1/2$, $\mathbf{v}_1 = (1,-0.6)^T$ and $\lambda_2 = -2$, $\mathbf{v}_2 = (0,1)^T$. The eigenvalues have opposite sign. Therefore, $(0,2)$ is a saddle, which is unstable. At $(1.5,0)$,

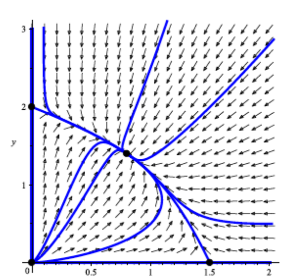
$$\mathbf{J}(1.5,0) = \begin{pmatrix} -1.5 & -0.75 \\ 0 & 0.875 \end{pmatrix}.$$

The associated eigenvalues and eigenvectors are $\lambda_1 = -1.5$, $\mathbf{v}_1 = (1,0)^T$ and $\lambda_2 = 0.875$, $\mathbf{v}_2 = (-0.31579,1)^T$. The eigenvalues are opposite sign. Therefore, $(1.5,0)$ is a saddle, which is unstable. At $(0.8,1.4)$,

$$\mathbf{J}(0.8,1.4) = \begin{pmatrix} -0.8 & -0.4 \\ -1.05 & -1.4 \end{pmatrix}.$$

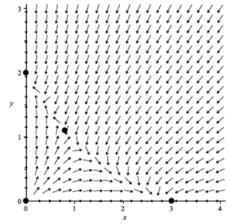
The associated eigenvalues and eigenvectors are $\lambda_1 = (-11+\sqrt{51})/10$, $\mathbf{v}_1 = (1,(3-\sqrt{51})/4)^T$ and $\lambda_2 = (-11-\sqrt{51})/10$, $\mathbf{v}_2 = (1,(3+\sqrt{51})/4)^T$. The eigenvalues are negative. Therefore, $(0.8,1.4)$ is a stable node, which is asymptotically stable.

(d,e)



(f) Except for initial conditions lying on the coordinate axes, all trajectories converge to the stable node $(0.8,1.4)$.

3.(a)



(b) The critical points are solutions of the system

$$\begin{aligned} x(1.5 - 0.5x - y) &= 0 \\ y(2 - y - 1.125x) &= 0. \end{aligned}$$

The four critical points are $(0,0)$, $(0,2)$, $(3,0)$, and $(4/5, 11/10)$.

(c) The Jacobian matrix is

$$\mathbf{J}(x,y) = \begin{pmatrix} 3/2 - x - y & -x \\ -1.125y & 2 - 2y - 1.125x \end{pmatrix}.$$

At $(0,0)$,

$$\mathbf{J}(0,0) = \begin{pmatrix} 3/2 & 0 \\ 0 & 2 \end{pmatrix}.$$

The associated eigenvalues and eigenvectors are $\lambda_1 = 3/2$, $\mathbf{v}_1 = (1,0)^T$ and $\lambda_2 = 2$, $\mathbf{v}_2 = (0,1)^T$. The eigenvalues are positive. Therefore, the origin is an unstable node. At $(0,2)$,

$$\mathbf{J}(0,2) = \begin{pmatrix} -1/2 & 0 \\ -9/4 & -2 \end{pmatrix}.$$

The associated eigenvalues and eigenvectors are $\lambda_1 = -1/2$, $\mathbf{v}_1 = (1, -3/2)^T$ and $\lambda_2 = -2$, $\mathbf{v}_2 = (0,1)^T$. The eigenvalues are both negative. Therefore, $(0,2)$ is a a stable node, which is asymptotically stable. At $(3,0)$,

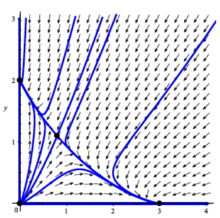
$$\mathbf{J}(3,0) = \begin{pmatrix} -3/2 & -3 \\ 0 & -11/8 \end{pmatrix}.$$

The associated eigenvalues and eigenvectors are $\lambda_1 = -3/2$, $\mathbf{v}_1 = (1,0)^T$ and $\lambda_2 = -11/8$, $\mathbf{v}_2 = (-24, 1)^T$. The eigenvalues are both negative. Therefore, this critical point is a stable node, which is asymptotically stable. At $(4/5, 11/10)$,

$$\mathbf{J}(4/5, 11/10) = \begin{pmatrix} -2/5 & -4/5 \\ -99/80 & -11/10 \end{pmatrix}.$$

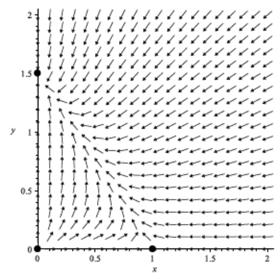
The associated eigenvalues and eigenvectors are $\lambda_1 = -3/4 + \sqrt{445}/20$, $\mathbf{v}_1 = (1, (7 - \sqrt{445})/16)^T$ and $\lambda_2 = -3/4 - \sqrt{445}/20$, $\mathbf{v}_2 = (0, (7 + \sqrt{445})/16)^T$. The eigenvalues are of opposite sign. Therefore, $(4/5, 11/10)$ is a saddle, which is unstable.

(d,e)



(f) Trajectories approaching the critical point $(4/5, 11/10)$ form a separatrix. Solutions on either side of the separatrix approach either $(3,0)$ or $(0,2)$.

5.(a)



(b) The critical points are solutions of the system

$$\begin{aligned} x(1-x-y) &= 0 \\ y(1.5-y-x) &= 0. \end{aligned}$$

The three critical points are $(0,0)$, $(0,3/2)$, and $(1,0)$.

(c) The Jacobian matrix is

$$\mathbf{J}(x,y) = \begin{pmatrix} 1-2x-y & -x \\ -y & 1.5-2y-x \end{pmatrix}.$$

At $(0,0)$,

$$\mathbf{J}(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & 1.5 \end{pmatrix}.$$

The associated eigenvalues and eigenvectors are $\lambda_1 = 1$, $\mathbf{v}_1 = (1,0)^T$ and $\lambda_2 = 1.5$, $\mathbf{v}_2 = (0,1)^T$. The eigenvalues are positive. Therefore, the origin is an unstable node. At $(0,3/2)$,

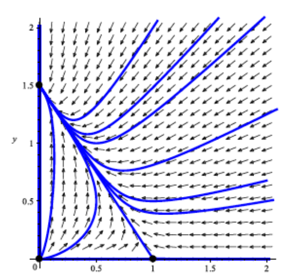
$$\mathbf{J}(0,3/2) = \begin{pmatrix} -1/2 & 0 \\ -3/2 & -3/2 \end{pmatrix}.$$

The associated eigenvalues and eigenvectors are $\lambda_1 = -1/2$, $\mathbf{v}_1 = (1,-3/2)^T$ and $\lambda_2 = -3/2$, $\mathbf{v}_2 = (0,1)^T$. The eigenvalues are both negative. Therefore, $(0,3/2)$ is a stable node, which is asymptotically stable. At $(1,0)$,

$$\mathbf{J}(1,0) = \begin{pmatrix} -1 & -1 \\ 0 & 1/2 \end{pmatrix}.$$

The associated eigenvalues and eigenvectors are $\lambda_1 = -1$, $\mathbf{v}_1 = (1,0)^T$ and $\lambda_2 = 1/2$, $\mathbf{v}_2 = (1,-3/2)^T$. The eigenvalues are of opposite sign. Therefore, this critical point is a saddle, which is unstable.

(d,e)



(f) All trajectories (not starting on the x-axis) converge to the stable node $(0,1.5)$.

10.(a) The critical points are solutions to the system:

$$\begin{aligned} -y &= 0 \\ -\gamma y - x(x-0.15)(x-3) &= 0. \end{aligned}$$

Setting $y = 0$, the second equation becomes $x(x-0.15)(x-3) = 0$. Therefore, the critical points are $(0,0)$, $(0.15,0)$, and $(3,0)$. The Jacobian matrix is

$$\mathbf{J}(x,y) = \begin{pmatrix} 0 & -1 \\ -3x^2 + 6.3x - 0.45 & -\gamma \end{pmatrix}.$$

At $(0,0)$,

$$\mathbf{J}(0,0) = \begin{pmatrix} 0 & -1 \\ -0.45 & -\gamma \end{pmatrix}.$$

The associated eigenvalues are $\lambda = -\gamma/2 \pm \sqrt{\gamma^2 + 1.8}/2$. Since the eigenvalues have opposite sign, the origin is a saddle, which is unstable. At $(0.15, 0)$,

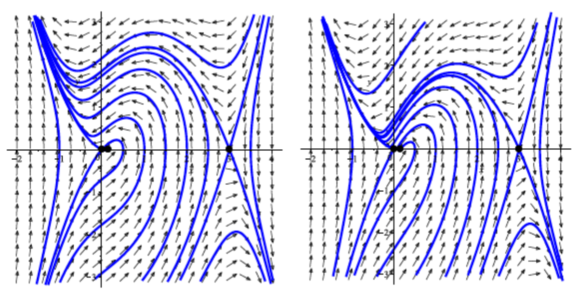
$$\mathbf{J}(0.15,0) = \begin{pmatrix} 0 & -1 \\ 0.4275 & -\gamma \end{pmatrix}.$$

The associated eigenvalues are $\lambda = -\gamma/2 \pm \sqrt{\gamma^2 - 1.71}/2$. If $\gamma^2 - 1.71 \geq 0$, then the eigenvalues are real. Since $\lambda_1 \lambda_2 = 0.4275$, both eigenvalues will have the same sign. Therefore, the critical point is a node with its stability dependent on the sign of γ. If $\gamma^2 - 1.71 < 0$, then the eigenvalues are complex conjugates. In that case, the critical point $(0.15, 0)$ is a spiral, with its stability dependent on the sign of γ. At $(3,0)$,

$$\mathbf{J}(3,0) = \begin{pmatrix} 0 & -1 \\ -8.55 & -\gamma \end{pmatrix}.$$

The associated eigenvalues are $\lambda = -\gamma/2 \pm \sqrt{\gamma^2 + 34.2}/2$. Since the eigenvalues have opposite sign, $(3,0)$ is a saddle, which is unstable.

(b)



(c) Based on the phase portraits above, we can see that the value of γ is above 1.5. Numerical experiments show that the required value is about $\gamma \approx 1.90$.