# Phase Plane Tutorial

This tutorial will walk you through how to use the function plotPhasePlane.m function.

The function plotPhasePlane.m will draw the phase portrait for a 2x2 systems of linear, constant-coefficient, homogeneous differential equations, like

$$\begin{bmatrix} \frac{dy_1}{dt} \\ \frac{dy_2}{dt} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

The function has inputs:

A - the 2x2 matrix of coefficients

yBounds - a vector with two entries holding the minimum and maximum values of $y$ for the state space

tBounds - a vector with two entries holding the minimum and maximum values of $t$ for plotting trajectories (you may want to include negative $t$)

M - the number of trajectories you want to plot

The output of plotPhasePlane.m is a phase portrait that has the straight line solutions given by the eigenvalues as black dashed lines and the trajectores as colored curves. Note that it will not plot any eigenvectors when eigenvalues are complex (i.e., there are no straight line solutions)

```
clear all
% define the matrix of coefficients

 %Richardson Stable Arms Race
```

## Richardson Stable Arms Race

```
  A = [-5 4;3 -4]
```

```
A = 2×2
    -5     4
     3    -4
```

```
 RHS = [1;2];

 B = rref([A -RHS]);

 eqSoln = B(:,end)
```

```
eqSoln = 2×1
    1.5000
    1.6250
```

```
% find the eigenvalues and eigenvectors of the matrix
[X D] = eig(sym(A))
```

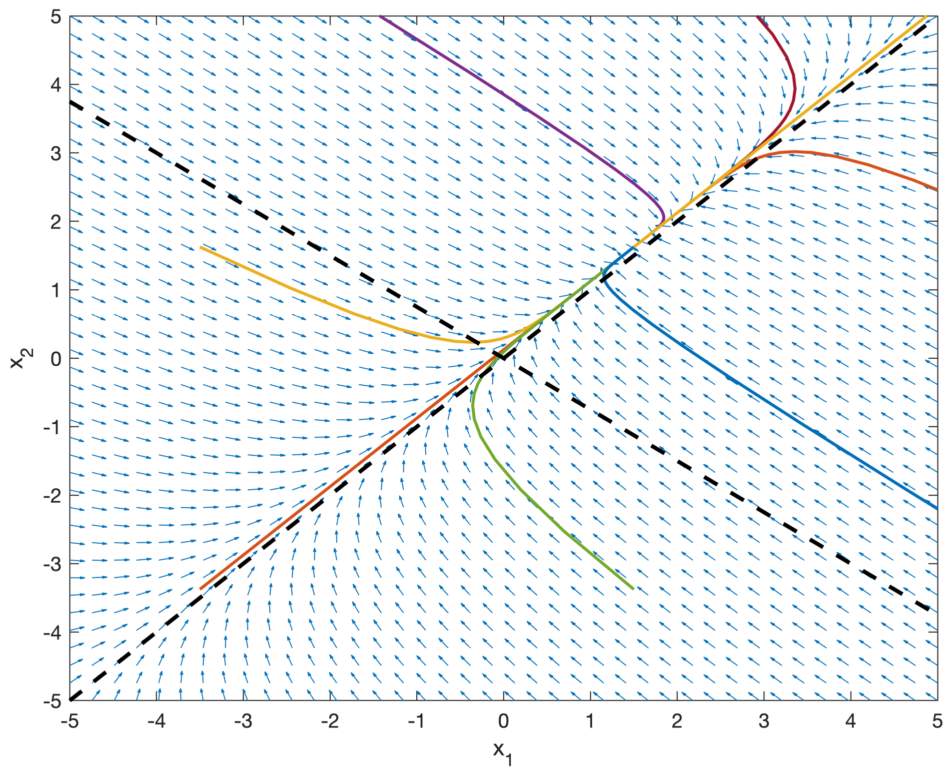
X =

$$\begin{pmatrix} -\frac{4}{3} & 1 \\ 1 & 1 \end{pmatrix}$$

D =

$$\begin{pmatrix} -8 & 0 \\ 0 & -1 \end{pmatrix}$$

```
% state the bounds for y and t
yBounds = [-5  5];  % [ymin ymax]
tBounds = [0, 50]; % [tmin tmax]
M = 3; % number of trajectories to plot
plotPhasePlane(A, yBounds, tBounds,M,RHS,eqSoln)
```



## Richardson Unstable Arms Race

```
A = [-9 11; 12 -8 ]
```

```
A = 2×2
    -9    11
    12    -8
```

```
RHS = [1;2];
B = rref([A -RHS]);
eqSoln = B(:,end)
```

```
eqSoln = 2×1
   -0.5000
   -0.5000
```

```
[X D] = eig(sym(A))
```

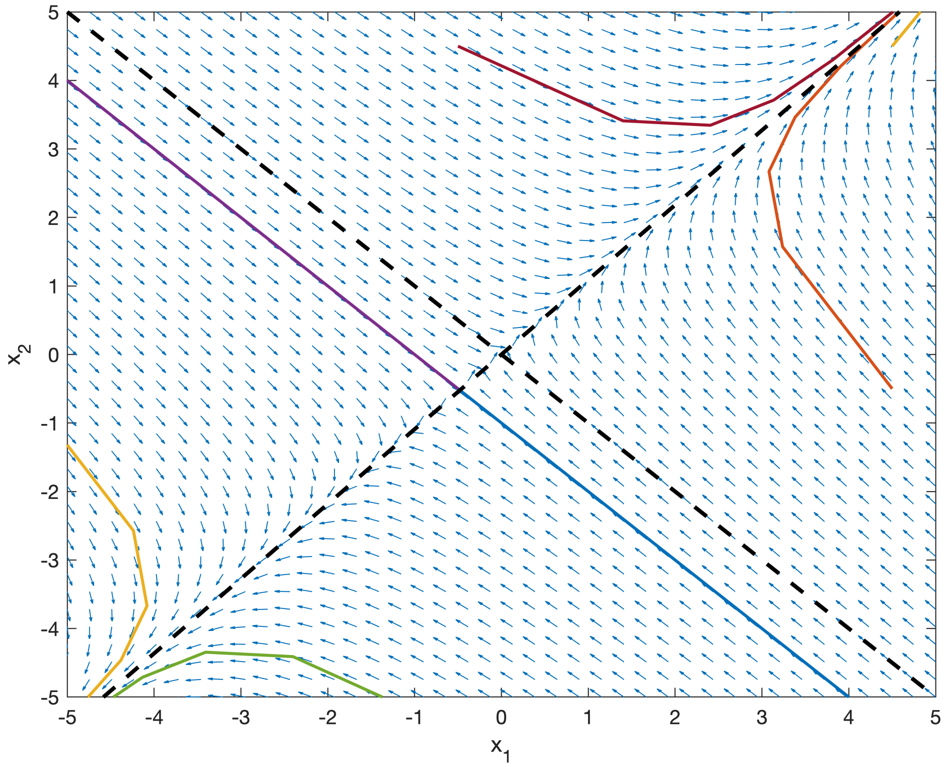
X =

$$\begin{pmatrix} -1 & \frac{11}{12} \\ 1 & 1 \end{pmatrix}$$

D =

$$\begin{pmatrix} -20 & 0 \\ 0 & 3 \end{pmatrix}$$

```
plotPhasePlane(A, yBounds, tBounds,M,RHS,eqSoln)
```




## Zero Eigenvalue Example

```
A = [-3 6; 4 -8]
```

```
A = 2×2
    -3     6
     4    -8
```

```
RHS = [1;2];
B = rref([A -RHS]);
eqSoln = B(:,end)
```

```
eqSoln = 2×1
     0
     1
```

```
[X D] = eig(sym(A))
```

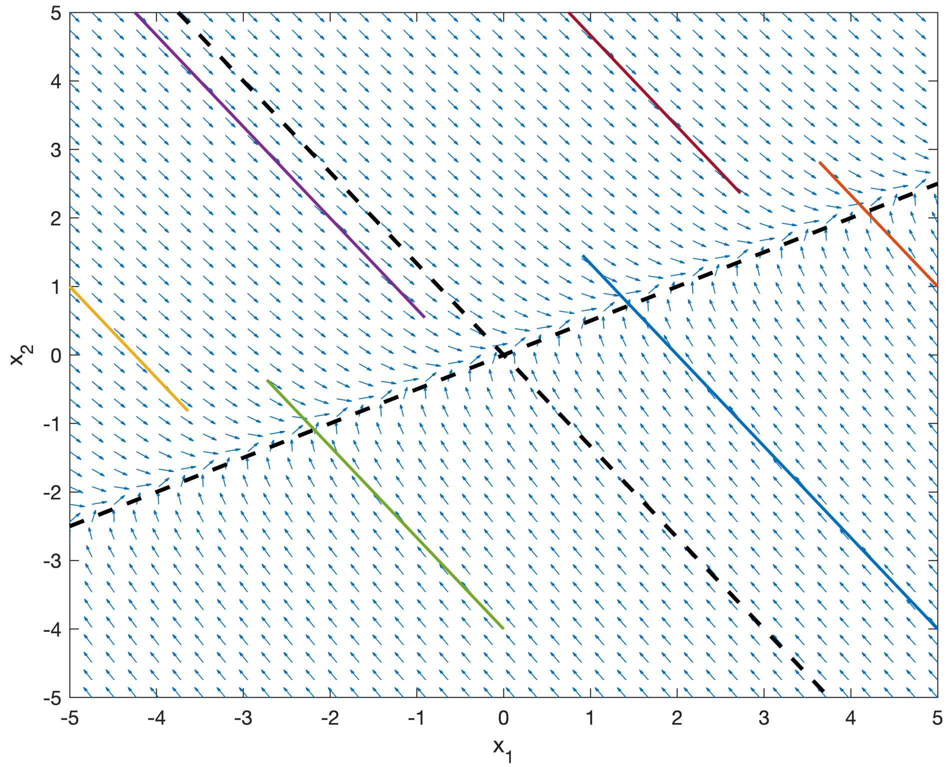
X =

$$\begin{pmatrix} 2 & -\frac{3}{4} \\ 1 & 1 \end{pmatrix}$$

D =

$$\begin{pmatrix} 0 & 0 \\ 0 & -11 \end{pmatrix}$$

```
plotPhasePlane(A, yBounds, tBounds,M,RHS,eqSoln)
```




## A Complex Eigennvalue Exampe

```
A = [0 1; -2 0];
RHS = [1;2];
B = rref([A -RHS]);
eqSoln = B(:,end)
```

```
eqSoln = 2×1
     1
    -1
```

```
[X D] = eig(sym(A))
```

```
X =
```

$$\begin{pmatrix} \frac{\sqrt{2}\,\mathrm{i}}{2} & -\frac{\sqrt{2}\,\mathrm{i}}{2} \\ 1 & 1 \end{pmatrix}$$

D =

$$\begin{pmatrix} -\sqrt{2}\,\mathrm{i} & 0 \\ 0 & \sqrt{2}\,\mathrm{i} \end{pmatrix}$$

```
plotPhasePlane(A, yBounds, tBounds,M,RHS,eqSoln)
```