Solving Systems of First Order Linear Differential Equations in MATLAB

See https://www.mathworks.com/help/symbolic/solve-a-system-of-differential-equations.html for more details

A Stable Arms Race Example

```
syms x(t) y(t)

ode1 = diff(x) == -5*x + 4*y + 10;
ode2 =diff(y) == 3*x - 4*y + 3;
odes = [ode1; ode2 ]
```

```
odes(t) =
```

$$\begin{pmatrix} \frac{\partial}{\partial t}\, x(t) = 4\, y(t) - 5\, x(t) + 10 \\ \frac{\partial}{\partial t}\, y(t) = 3\, x(t) - 4\, y(t) + 3 \end{pmatrix}$$

```
S =dsolve(odes)
```

```
S = struct with fields:
    y: [1×1 sym]
    x: [1×1 sym]
```

```
xSol(t) = S.x
```

```
xSol(t) =
```

$$\mathrm{e}^{-t}\left(C_{42} + 6\,\mathrm{e}^{t}\right) - \frac{4\,\mathrm{e}^{-8\,t}\left(C_{41} - \frac{3\,\mathrm{e}^{8\,t}}{8}\right)}{3}$$

```
ySol(t) = S.y
```

```
ySol(t) =
```

$$\mathrm{e}^{-t}\left(C_{42} + 6\,\mathrm{e}^{t}\right) + \mathrm{e}^{-8\,t}\left(C_{41} - \frac{3\,\mathrm{e}^{8\,t}}{8}\right)$$

```
cond1 = x(0) == 0;
cond2 = y(0) == 10;
```

```
conds = [cond1; cond2]
```

```
conds =
```

$$\begin{pmatrix} x(0) = 0 \\ y(0) = 10 \end{pmatrix}$$

```
[xSol(t), ySol(t)]= dsolve(odes, conds)
```

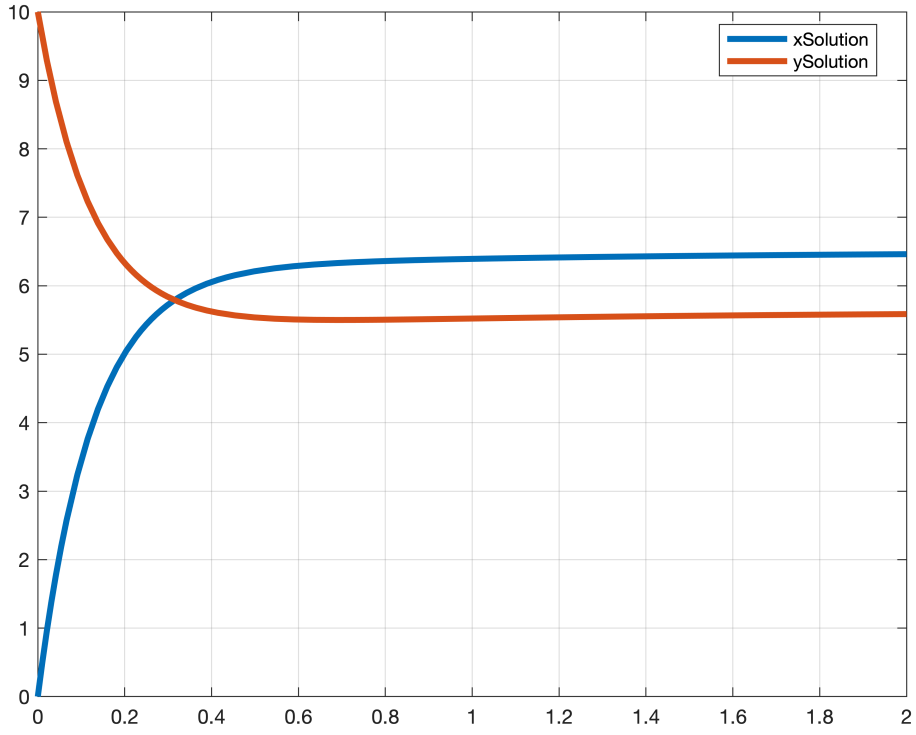
```
xSol(t) =
```

$$\mathrm{e}^{-t}\left(6\,\mathrm{e}^{t} - \frac{2}{7}\right) + \frac{4\,\mathrm{e}^{-8t}\left(\frac{3\,\mathrm{e}^{8t}}{8} - \frac{261}{56}\right)}{3}$$

```
ySol(t) =
```

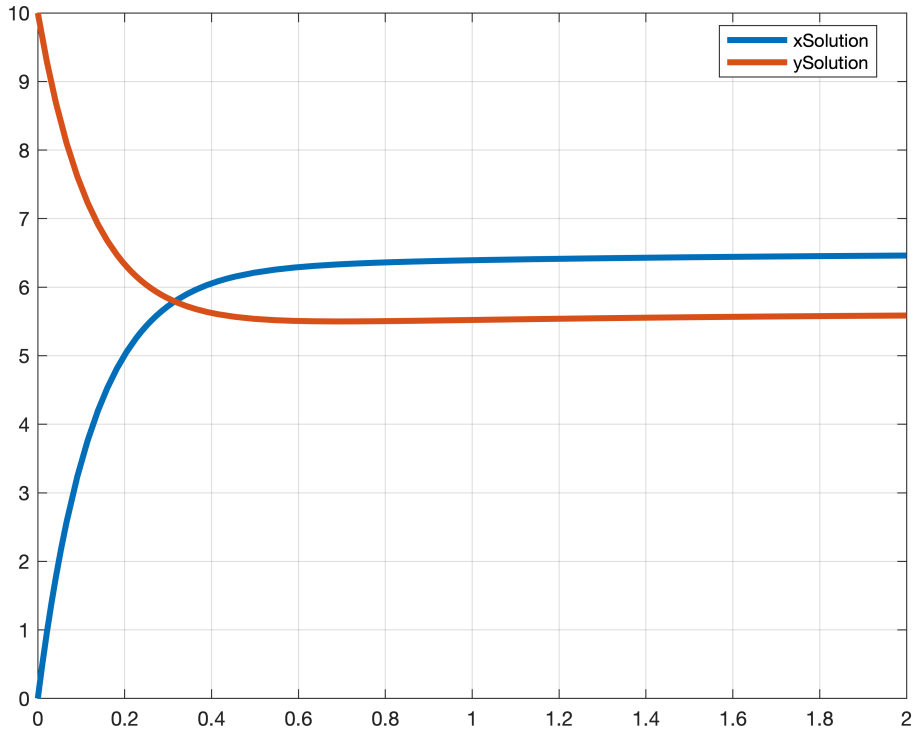
$$\mathrm{e}^{-t}\left(6\,\mathrm{e}^{t} - \frac{2}{7}\right) - \mathrm{e}^{-8t}\left(\frac{3\,\mathrm{e}^{8t}}{8} - \frac{261}{56}\right)$$

```
clf
fplot(xSol, [0 2], 'linewidth', 3)
hold on
fplot( ySol, [0 2], 'linewidth', 3)
grid on
legend( 'xSolution', 'ySolution', 'Location', 'best')
hold on
```



An Example With Complex Eigenvalues

```
hold off
```




```
ode1 = diff(x) == -.5*x + 1*y;
ode2 =diff(y) == -1*x - .5*y + 3;
odes = [ode1; ode2 ]
```

odes(t) =

$$\begin{pmatrix} \frac{\partial}{\partial t}\, x(t) = y(t) - \frac{x(t)}{2} \\ \frac{\partial}{\partial t}\, y(t) = 3 - \frac{y(t)}{2} - x(t) \end{pmatrix}$$

```
S =dsolve(odes)
```

```
S = struct with fields:
    y: [1×1 sym]
    x: [1×1 sym]
```

```
xSol(t) = S.x
```

xSol(t) =

$$e^{-\frac{t}{2}}\cos(t)\left(C_{54}+\frac{12\,e^{t/2}\left(\cos(t)-\frac{\sin(t)}{2}\right)}{5}\right)+e^{-\frac{t}{2}}\sin(t)\left(C_{53}+\frac{12\,e^{t/2}\left(\frac{\cos(t)}{2}+\sin(t)\right)}{5}\right)$$

```
ySol(t) = S.y
```

ySol(t) =

$$e^{-\frac{t}{2}}\cos(t)\left(C_{53}+\frac{12\,e^{t/2}\left(\frac{\cos(t)}{2}+\sin(t)\right)}{5}\right)-e^{-\frac{t}{2}}\sin(t)\left(C_{54}+\frac{12\,e^{t/2}\left(\cos(t)-\frac{\sin(t)}{2}\right)}{5}\right)$$

```
cond1 = x(0) == 0;
cond2 = y(0) == 10;
conds = [cond1; cond2]
```

conds =

$$\begin{pmatrix} x(0)=0 \\ y(0)=10 \end{pmatrix}$$

```
[xSol(t), ySol(t)]= dsolve(odes, conds)
```

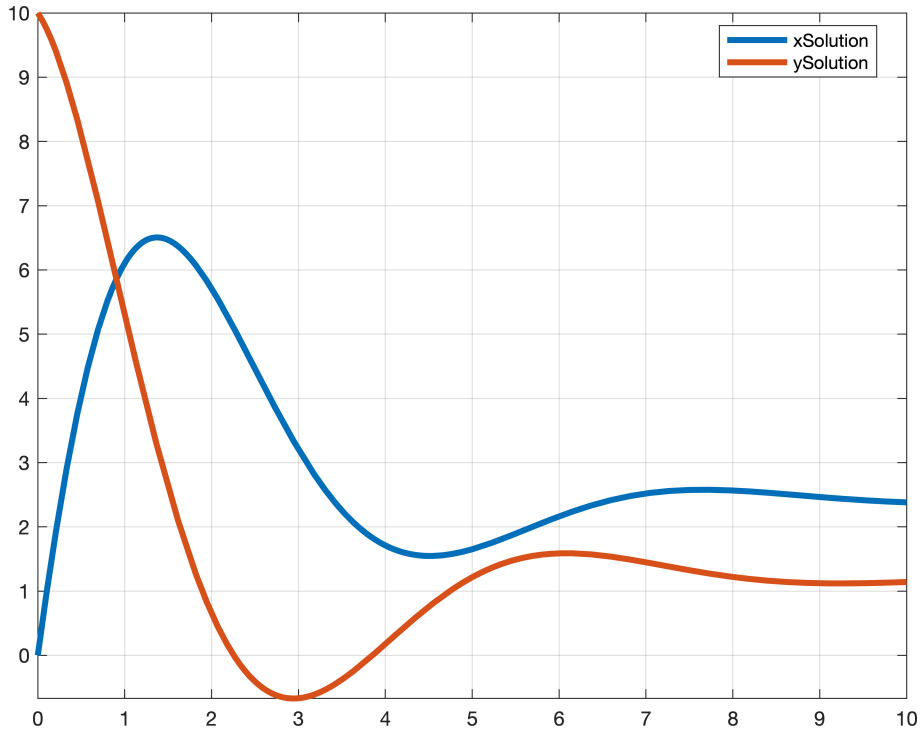
xSol(t) =

$$\frac{44\sin(t)}{5\sqrt{e^t}}-\frac{12\cos(t)}{5\sqrt{e^t}}+\frac{12}{5}$$

ySol(t) =

$$\frac{44\cos(t)}{5\sqrt{e^t}}+\frac{12\sin(t)}{5\sqrt{e^t}}+\frac{6}{5}$$

```
clf
fplot(xSol, [0 10], 'linewidth', 3)
hold on
fplot( ySol, [0 10], 'linewidth', 3)
grid on
legend( 'xSolution', 'ySolution', 'Location', 'best')
```


xSolution
ySolution
10
9
8
7
6
5
4
3
2
1
0
0
1
2
3
4
5
6
7
8
9
10