MATH 226

Modeling With First Order Differential Equations

2. A tank initially contains 200 L of pure water. A mixture containing a concentration of γ g/L of salt enters the tank at a rate of 4 L/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t. Also find the limiting amount of salt in the tank as $t \to \infty$.

7. An outdoor swimming pool loses 0.05% of its water volume every day it is in use, due to losses from evaporation and from excited swimmers who splash water. A system is available to continually replace water at a rate of G gallons per day of use.

(a) Find an expression, in terms of G, for the equilibrium volume of the pool. Sketch a few graphs for the volume $V(t)$, including all possible types of solutions.

(b) If the pool volume is initially 1% above its equilibrium value, find an expression for $V(t)$.

(c) What is the replacement rate G required to maintain 12,000 gal of water in the pool?

13. A recent college graduate borrows \$100,000 at an interest rate of 9% to purchase a condominium. Anticipating steady salary increases, the buyer expects to make payments at a monthly rate of $800(1 + t/120)$, where t is the number of months since the loan was made.

(a) Assuming that this payment schedule can be maintained, when will the loan be fully paid?
(b) Assuming the same payment schedule, how large a loan could be paid off in exactly 20 years?

25. A skydiver weighing 180 lb (including equipment) falls vertically downward from an altitude of 5,000 ft and opens the parachute after 10 s of free fall. Assume that the force of air resistance is $0.75|v|$ when the parachute is closed and $12|v|$ when the parachute is open, where the velocity v is measured in feet per second.

(a) Find the speed of the skydiver when the parachute opens.
(b) Find the distance fallen before the parachute opens.
(c) What is the limiting velocity v_L after the parachute opens?
(d) Determine how long the skydiver is in the air after the parachute opens.
(e) Plot the graph of velocity versus time from the beginning of the fall until the skydiver reaches the ground.

20. Heat transfer from a body to its surroundings by radiation, based on the Stefan–Boltzmann law, is described by the differential equation

$$\frac{du}{dt} = -\alpha(u^4 - T^4), \tag{i}$$

where $u(t)$ is the absolute temperature of the body at time t, T is the absolute temperature of the surroundings, and α is a constant depending on the physical parameters of the body. However, if u is much larger than T, then solutions of Eq. (i) are well approximated by solutions of the simpler equation

$$\frac{du}{dt} = -\alpha u^4. \tag{ii}$$

Suppose that a body with initial temperature 2000 K is surrounded by a medium with temperature 300 K and that $\alpha = 2.0 \times 10^{-12}\ \mathrm{K}^{-3}/\mathrm{s}$.

(a) Determine the temperature of the body at any time by solving Eq. (ii).

(b) Plot the graph of u versus t.

(c) Find the time τ at which $u(\tau) = 600$, that is, twice the ambient temperature. Up to this time, the error in using Eq. (ii) to approximate the solutions of Eq. (i) is no more than 1%.