MATH 226: Differential Equations

Class 13: March 14, 2022

MATLAB Worksheet

Linear Algebra Computations with *Maple*

Linear Algebra Computations with MATLAB

Graded Project 1

Announcements

- Exam 1
  - Tonight
  - 7 PM - ? (No Time Limit)
  - Axinn 229
  - No Calculators, Books, Notes, Smart Phones, etc.
  - Focus on Material in Chapters 1 and 2

Exam 1 Tips

- Read Overall Directions on Cover Page
- Read Each Problem Statement Carefully
- Pick "Easy" Problem To Do First
- Show All Intermediate Steps
- Include Explanations
- Pay Attention to Units
- Double Check Your Work

Mathematician of the Week

Vijay Kumar Patodi

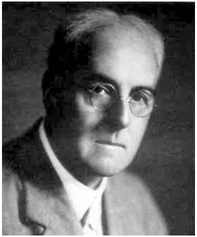
March 12, 1945 –December 21, 1976

Brilliant work at young age in analysis and geometry.

Today's Topic

Analysis of
The Richardson Arms Race Model

Richardson Arms Race Model



Lewis F. Richardson

1881 – 1953

$x(t)$ = **Arms Expenditure of Blue Nation**

$y(t)$ = **Arms Expenditure of Red Nation**

$$x' = ay - mx + r$$

$$y' = bx - ny + s$$

where a, b, m, n are positive constants while r and s are constants.

Structure: $\vec{X} = A\vec{X} + \vec{b}$ or $\mathbf{x'} = A\mathbf{x} + \mathbf{b}$

$$x' = ay - mx + r$$

$$y' = bx - ny + s$$

where a, b, m, n are positive constants while r and s are constants.

Find stable Lines L and L' where $x' = 0$ and $y' = 0$.

$$L : y = \frac{m}{a}x - \frac{r}{a}$$

$$L' : y = \frac{b}{n}x + \frac{s}{n}$$

Determine the Stable Point (x^*, y^*) where lines **L** and **L'** intersect.

$$ay^* - mx^* + r = 0, \; bx^* - ny^* + s = 0$$

$$x' = ay - mx + r$$

$$y' = bx - ny + s$$

$$ay^* - mx^* + r = 0, \;\; bx^* - ny^* + s = 0$$

Make Change of Variable $X = x - x^*, \;\; Y = y - y^*$

Then

$$X' = x' = a(Y + y^*) - m(X + x^*) + r = aY - mX + (ay^* - mx^* + r)$$

$$= aY - mX + 0 = aY - mX$$

Similarly, $Y' = bX - nY$

Write system as

$$X' = aY - mX$$

$$Y' = bX - nY$$

We transform

$$x' = ay - mx + r$$
$$y' = bx - ny + s$$

a nonhomgeneous system into

$$X' = -mX + aY$$
$$Y' = bX - nY$$

a homogeneous system.

$$\mathbf{X'} = A\,\mathbf{X}$$

where

$$A = \begin{bmatrix} -m & a \\ b & -n \end{bmatrix}$$

We can solve by finding eigenvalues and eigenvectors of A

$$\mathbf{X'} = A\,\mathbf{X}$$

where

$$A = \begin{bmatrix} -m & a \\ b & -n \end{bmatrix} \text{ has solution}$$

$$\alpha e^{\lambda t}\vec{v} + \beta e^{\mu t}\vec{w}$$

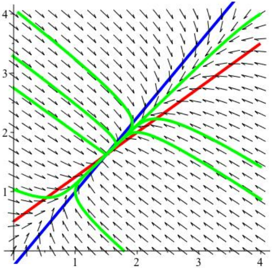
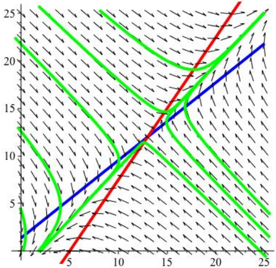
where α and β are arbitrary constants
λ is an eigenvalue of A with associated eigenvector $\vec{v}$ and
$\mu \neq \lambda$ is an eigenvalue of A with associated eigenvector $\vec{w}$.

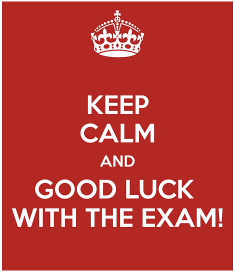
The solution of the original system is then

$$\alpha e^{\lambda t}\vec{v} + \beta e^{\mu t}\vec{w} + \begin{bmatrix} x^* \\ y^* \end{bmatrix}$$

Two Particular Examples:

$x' = -5x + 4y + 1$ $y' = 3x - 4y + 2$	$x' = 11y - 9x - 15$ $y' = 12x - 8y - 60$
$(x^*, y^*) = (\frac{3}{2}, \frac{13}{8})$	$(x^*, y^*) = (13, 12)$
$A = \begin{bmatrix} -5 & 4 \\ 3 & -4 \end{bmatrix}$	$A = \begin{bmatrix} -9 & 11 \\ 12 & -8 \end{bmatrix}$
$\lambda = -1, \vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\lambda = 3, \vec{v} = \begin{bmatrix} 11 \\ 12 \end{bmatrix}$
$\mu = -8, \vec{w} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$	$\mu = -20, \vec{w} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
$\alpha e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \beta e^{-8t} \begin{bmatrix} -4 \\ 3 \end{bmatrix} + \begin{bmatrix} \frac{3}{2} \\ \frac{13}{8} \end{bmatrix}$	$\alpha e^{3t} \begin{bmatrix} 11 \\ 12 \end{bmatrix} + \beta e^{-20t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 13 \\ 12 \end{bmatrix}$

$x' = -5x + 4y + 1$ $y' = 3x - 4y + 2$	$x' = 11y - 9x - 15$ $y' = 12x - 8y - 60$
	


KEEP
CALM
AND
GOOD LUCK
WITH THE EXAM!


I KNOW YOU CAN DO IT!
THINK
POSITIVE