CLASS 8

HANDOUTS

NOTES ON ASSIGNMENT 7

ASSIGNMENT 8

NOTES ON SAMPLE EXAM 1

UNIFIED TREATMENT OF TANGENT LINES AND PLANES

Parametrized Surfaces in Maple (HANDOUTS folder)

ANNOUNCEMENTS

EXAM 1: Monday, 7PM –

NO TIME LIMIT, NO BOOKS, NOTES OR CALCULATORS

223A: AXN 229

223B: AXN 232

TANGENT PLANE TO GRAPH OF $f: \mathbb{R}^n \to \mathbb{R}^1$

POINT $(\vec{a}, f(\vec{a}))$ ON HYPERSURFACE

$n=2$ $T(\vec{x}) = f(\vec{a}) + (f_x(\vec{a}), f_y(\vec{a})) \cdot (\vec{x} - \vec{a})$

IN general

$$T(\vec{x}) = f(\vec{a}) + \nabla f(\vec{a}) \cdot (\vec{x} - \vec{a})$$

where $\nabla f(\vec{a}) = (f_{x_1}(\vec{a}), f_{x_2}(\vec{a}), \ldots, f_{x_n}(\vec{a}))$

Tangent Hyperplane.

$n=1$ ORDINARY TANGENT LINE

$n=2$ TANGENT PLANE

EXAMPLE $f(x,y,z) = \frac{x^2y}{z}$

note: $f: \mathbb{R}^3 \to \mathbb{R}^1$

GRAPH LIVES IN $\mathbb{R}^4$

FIND EQUATION of Tangent Hyperplane

IF $\vec{a} = (-3, 4, 2)$

$f_x \to 2xy/z$

$f_y \to x^2/z$

$f_z \to -x^2y/z^2$

$$\nabla f(x,y,z) = \left(\frac{2xy}{z}, \frac{x^2}{z}, -\frac{x^2y}{z^2} \right)$$

at $\vec{a} = (-3, 4, 2)$

$$f(\vec{a}) = \frac{(-3)^2 \cdot 4}{2} = 18$$

$$\nabla f(\vec{a}) = \left(\frac{(2)(-3)(4)}{2}, \frac{(-3)^2}{2}, -\frac{(-3)^2(4)}{2^2} \right)$$

$$= \left(-12, \tfrac{9}{2}, -9 \right)$$

Equation of Tangent PLANE

$$w = 18 + \left(-12, \tfrac{9}{2}, -9\right)(x+3, y-4, z-2)$$

CLAIRAUT'S THEOREM (EQUALITY of Mixed PARTIALS)

If f_{xy} and f_{yx} are CONTINUOUS at $\vec{a}$, then $f_{xy}(\vec{a}) = f_{yx}(\vec{a})$

ALEXIS CLAUDE CLAIRAUT

MAY 7, 1713 – MAY 17, 1765

PARAMETRIZED SURFACES

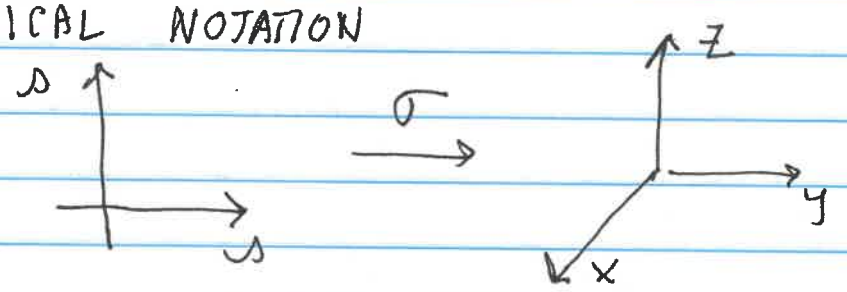
FUNCTION from $\mathbb{R}^2 \longrightarrow \mathbb{R}^3$

DOMAIN: PATCH IN PLANE

IMAGE: SURFACE IN SPACE

GRAPH: LIVES IN $\mathbb{R}^5$

TYPICAL NOTATION



Need for parametrizations

graph of $f: \mathbb{R}^1 \to \mathbb{R}^1$ IS CURVE IN $\mathbb{R}^2$

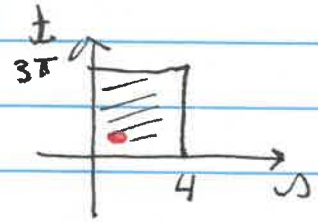
BUT NOT EVERY CURVE IS THE GRAPH OF A FUNCTION

SIMILARLY, graph of $f: \mathbb{R}^2 \to \mathbb{R}^1$ IS SURFACE IN $\mathbb{R}^3$

BUT NOT EVERY SURFACE IS GRAPH OF SUCH A FUNCTION.

Example $\sigma(s,t) = (s\cos t, s\sin t, t)$

$0 \le s \le 4, \quad 0 \le t \le 3\pi$



POINT $(1, \pi/4)$

$\sigma(1, \pi/4) = (\sqrt{2}/2, \sqrt{2}/2, \pi/4)$

$\sigma_s = (\cos t, \sin t, 0)$

$\sigma_t = (-s\sin t, s\cos t, 1)$

at $(1, \pi/4)$ $\quad \sigma(1, \frac{\pi}{4}) + \sigma_s(1, \pi/4)\, s + \sigma_t(1, \frac{\pi}{4})\, t$

PARAMETRIZED Tangent Plane

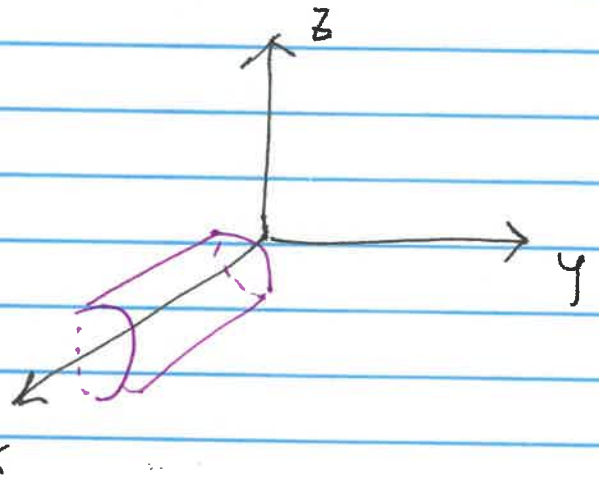
PARAMETRIZE CYLINDER.

$$y^2 + z^2 = 16$$

Let $x = s$

$y = 4 \cos t$

$z = 4 \sin t$

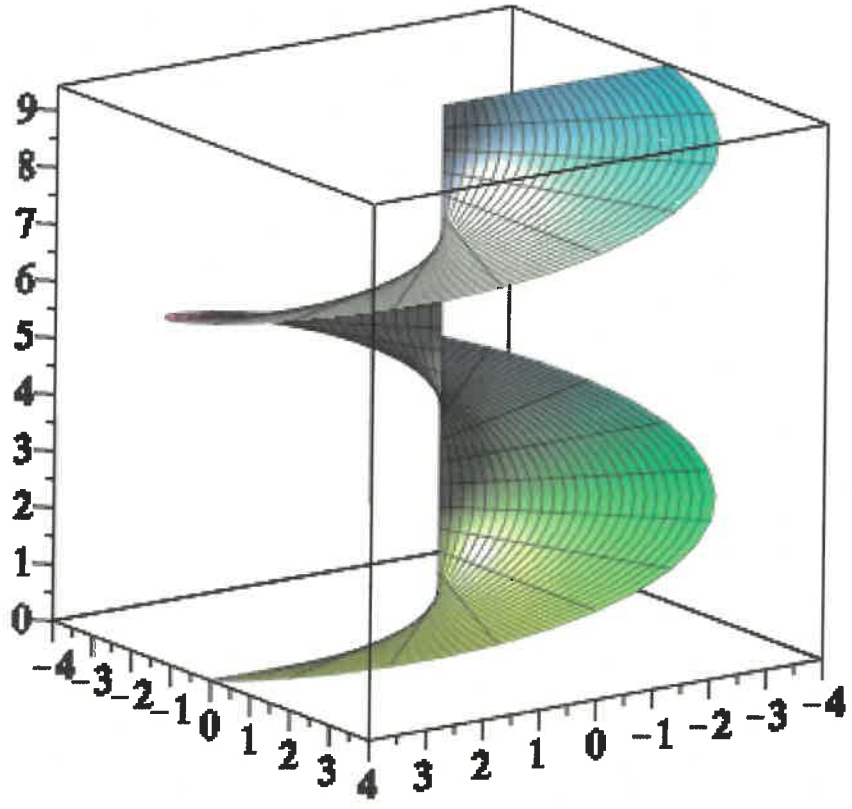


MATH 223

Parametrized Surfaces

$with(plots):$

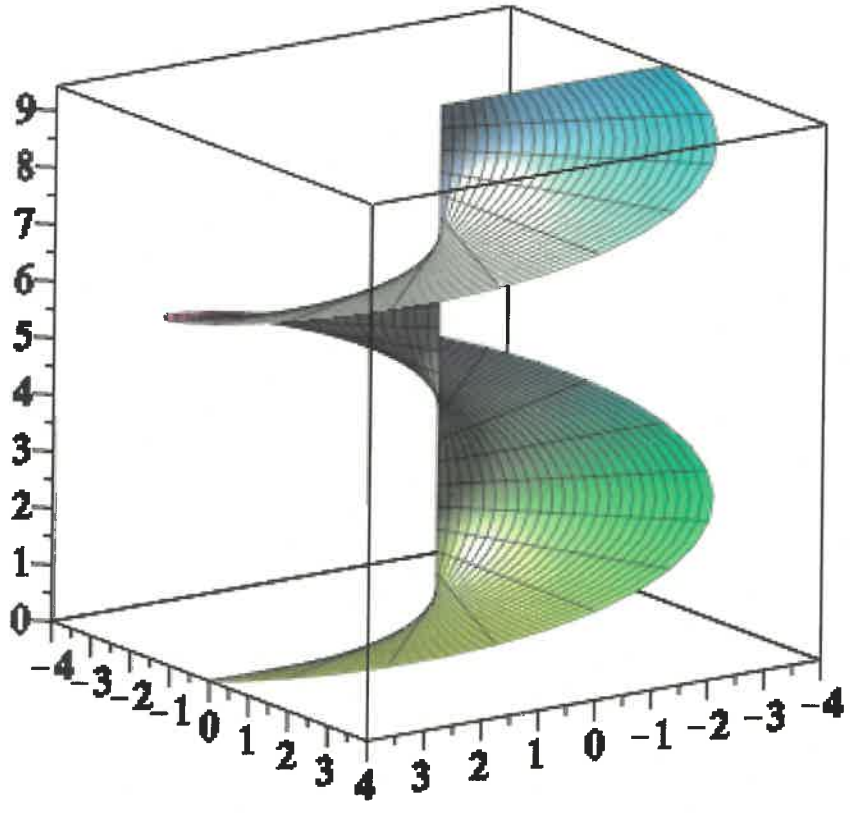
$plot3d([s \cdot \cos(t), s \cdot \sin(t), t], s = 0..4, t = 0..3 \cdot \pi)$



$f := (s, t) \to [\, s \cdot \cos(t), s \cdot \sin(t), t]$

$$f := (s, t) \mapsto [s\cos(t), s\sin(t), t] \tag{1}$$

$plot3d(f(s,t), s = 0..4, t = 0..3\cdot\pi)$



Parametrization of Tangent Plane

$f\left(1, \frac{\pi}{4}\right)$

$$\left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\pi}{4}\right] \tag{2}$$

$f(s, t)$

$$[s\cos(t), s\sin(t), t] \tag{3}$$

$fs := (s, t) \to [\, \cos(t), \sin(t), 0]$

$$fs := (s, t) \mapsto [\cos(t), \sin(t), 0] \tag{4}$$

$fs\left(1, \frac{\pi}{4}\right)$

$$\left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right] \tag{5}$$

$ft := (s, t) \to [-s\sin(t), s\cos(t), 1]$

$$ft := (s, t) \mapsto [-s\sin(t), s\cos(t), 1] \tag{6}$$

$ft\left(1, \frac{\pi}{4}\right)$

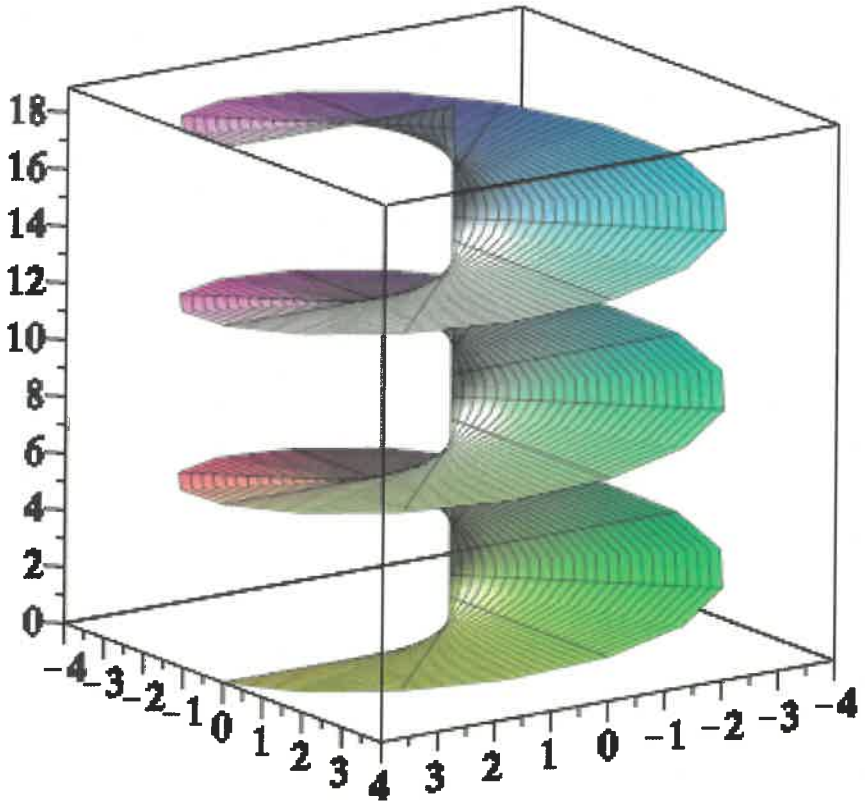
$$\left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1\right] \tag{7}$$

$T := f\left(1, \frac{\pi}{4}\right) + fs\left(1, \frac{\pi}{4}\right)\cdot s + ft\left(1, \frac{\pi}{4}\right)\cdot t$

$$T := \left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\pi}{4}\right] + \left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right]s + \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1\right]t \tag{8}$$

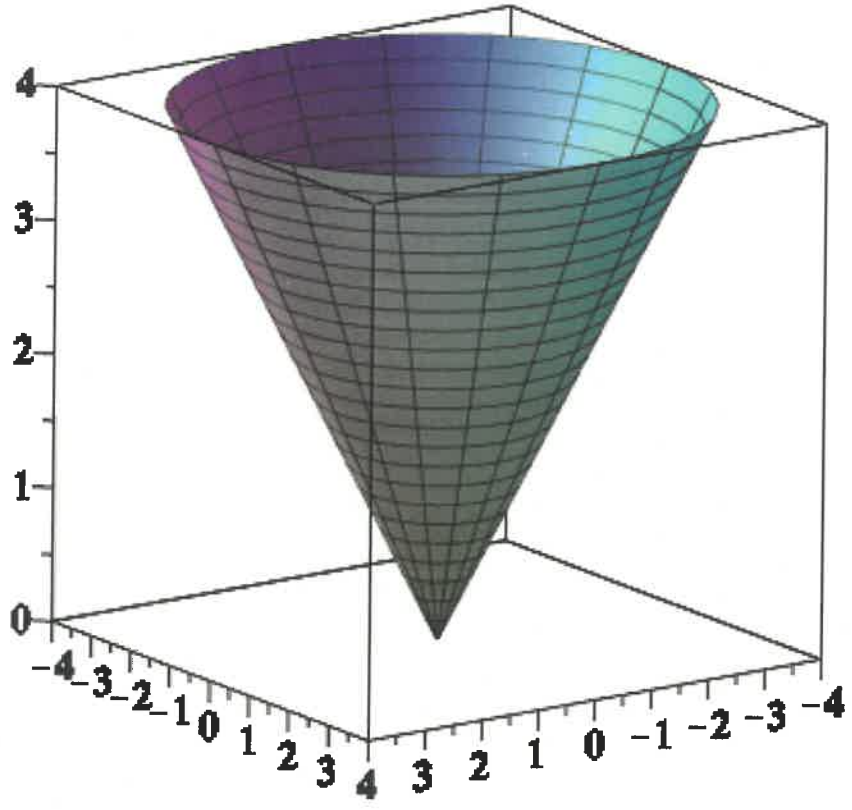
Extend the domain on t:

$plot3d([s \cdot \cos(t), s \cdot \sin(t), t], s = 0..4, t = 0..6 \cdot \pi)$



Replace t with s for 3rd Component

$plot3d([\, s \cdot \cos(t), s \cdot \sin(t), s \,], s = 0..4, t = 0..3 \cdot \pi)$



Parametrize Cylinder

$plot3d([s, 4 \cdot \cos(t), 4 \cdot \sin(t)], s = -6..5, t = 0..2 \cdot \pi)$