HANDOUTS

- NOTES ON ASSIGNMENT 6
- ASSIGNMENT 7
- SAMPLE EXAM

IN HANDOUTS (CLASS 7)

ANNOUNCEMENTS

- EXAM 1: Next Monday, 7 PM –
- 223A: AXINN 229
- 223B: AXINN 232

THIS WEEK

- PARTIAL DERIVATIVES
- PARAMETRIC SURFACES
- APPLICATIONS

PARTIAL DERIVATIVES

Setting: $f: \mathbb{R}^2 \to \mathbb{R}^1$

PARTIAL WITH RESPECT TO x at (a,b)

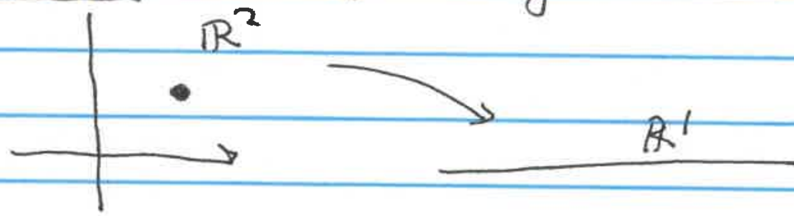
$$\frac{\partial f}{\partial x}(a,b) = f_x(a,b) = \lim_{t\to 0} \frac{f(a+t,b) - f(a,b)}{t}$$

PARTIAL WITH RESPECT TO y at (a,b)

$$\frac{\partial f}{\partial y}(a,b) = f_y(a,b) = \lim_{t\to 0} \frac{f(a,b+t) - f(a,b)}{t}$$

NOTE: BOTH OF THESE WILL BE NUMBERS

Example $f(x,y) = x^2 y$ at $(2,3)$



NOTE: VALUE OF FUNCTION $f(2,3) = 2^2 \cdot 3 = 12$

f_x $\quad \dfrac{f(2+t,3) - f(2,3)}{t} = \dfrac{(2+t)^2 \cdot 3 - 12}{t}$

$= \dfrac{(4+4t+t^2)\,3 - 12}{t} = \dfrac{12+12t+3t^2-12}{t}$

$= 12 + 3t$

$\lim_{t \to 0} (12 + 3t) = 12$ so $f_x(2,3) = 12$

f_y $\quad \dfrac{f(2,3+t) - f(2,3)}{t} = \dfrac{2^2(3+t) - 12}{t} = \dfrac{12+4t-12}{t} = 4$

so $\lim_{t \to 0} 4 = 4$ Hence $f_y(2,3) = 4$

WHAT'S GOING ON VISUALLY ?

RESTATE: $f(x,y) = x^2 y$

$\vec{a} = (2,3)$

Then $f(\vec{a}) = f(2,3) = 12$

$f_x(\vec{a}) = 12$

$f_y(\vec{a}) = 4$

We can write equation of a plane

$$Z = 12 + 12(x-2) + 4(y-3)$$

$$= f(2,3) + (12,4) \cdot (x-2, y-3)$$

$$= f(\vec{a}) + (12,4) \cdot [(x,y) - (2,3)]$$

$$= f(\vec{a}) + [f_x(a,b), f_y(a,b)] \cdot (\vec{x} - \vec{a})$$

$$= f(\vec{a}) + \nabla f(\vec{a}) \cdot (\vec{x} - \vec{a})$$

↳ gradient of f at $\vec{a}$

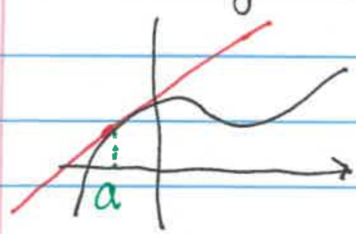
CAN ALSO WRITE as

$Z = 12 + 12x - 24 + 4y - 12$

$Z = 12x + 4y - 24$

ANALOGY:

Tangent Line for $f: \mathbb{R}^1 \to \mathbb{R}^1$



$$y = f(a) + f'(a)(x-a)$$

CALCULATIONS

f_x: Treat y as a constant } Apply Usual Rules of Differentiation

f_y: Treat x as a constant }

Example $\omega = f(x,y,z) = x^2 y + y^2 \ln z$

$f_x(x,y) = 2xy$

$f_y(x,y) = x^2 + 2y \ln z$

$f_z(x,y) = y^2/z$

HIGHER ORDER DERIVATIVES

f_{xx} f_{yx}

f_{xy} f_{yy}

Example $f(x,y) = x^2 y + \sin(x)\, e^y$

$f_x(x,y) = 2xy + e^y \cos x$

$f_{xx} = 2y - e^y \sin x$

$f_{xy} = 2x + e^y \cos x$

$f_y(x,y) = x^2 + e^y \sin x$

$f_{yx} = 2x + e^y \cos x$

$f_{yy} = e^y \sin x$

MATH 223

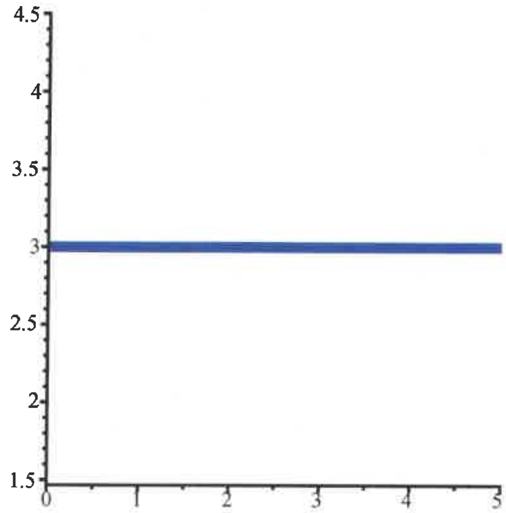
Partial Derivatives Example

$f(x,y) = x^2y$ at (2,3)

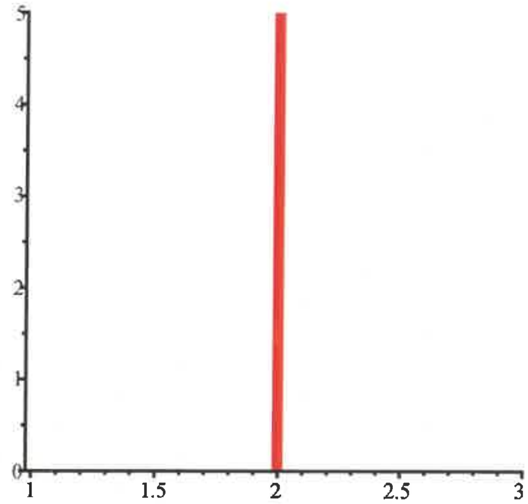
$f := (x, y) \rightarrow x^2 \cdot y$

$$f := (x, y) \mapsto x^2\, y \qquad (1)$$

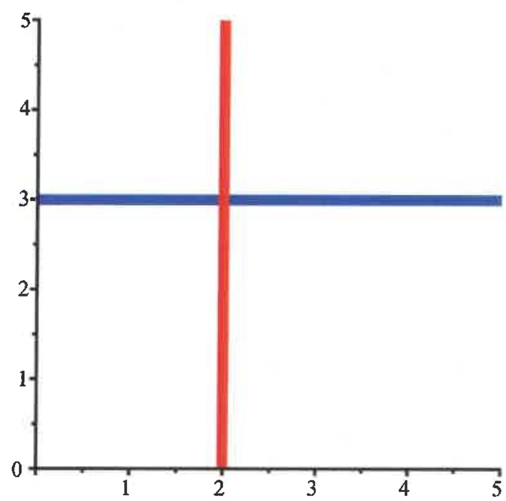
$L1 := plot([t, 3, t = 0\,..5], color = blue, thickness = 4)$



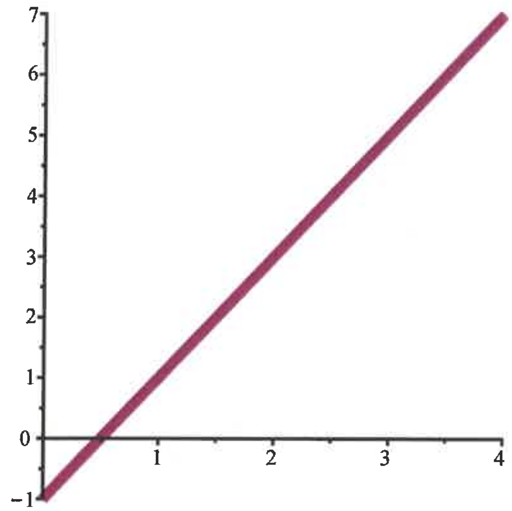
$L2 := plot([2, t, t = 0..5], color = red, thickness = 4)$



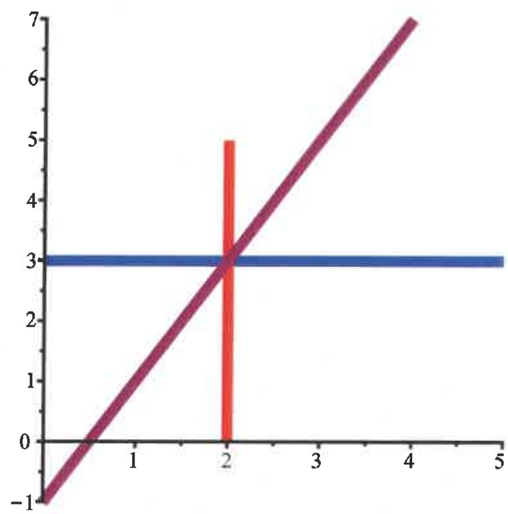
$with(plots) : display(L1, L2)$



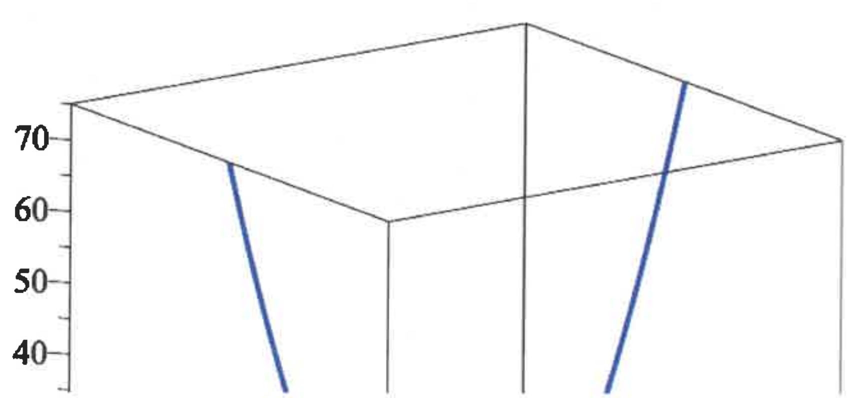
$L3 := plot([2 + t, 3 + 2\cdot t, t = -2..2], color = purple, thickness = 4)$



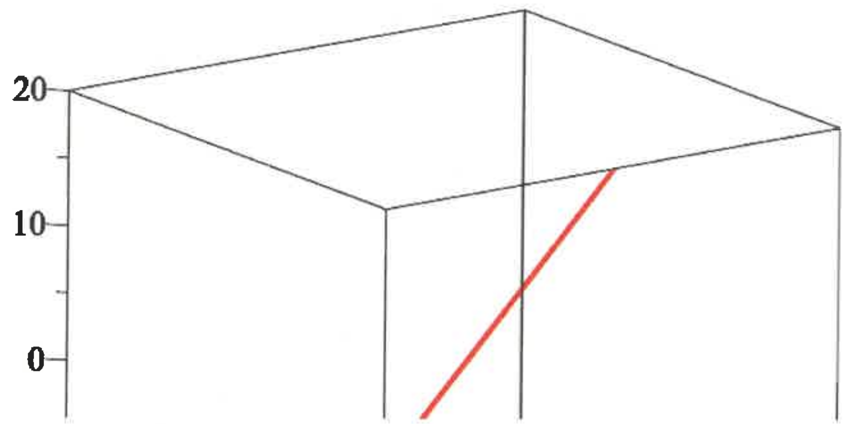
$display(L1, L2, L3)$



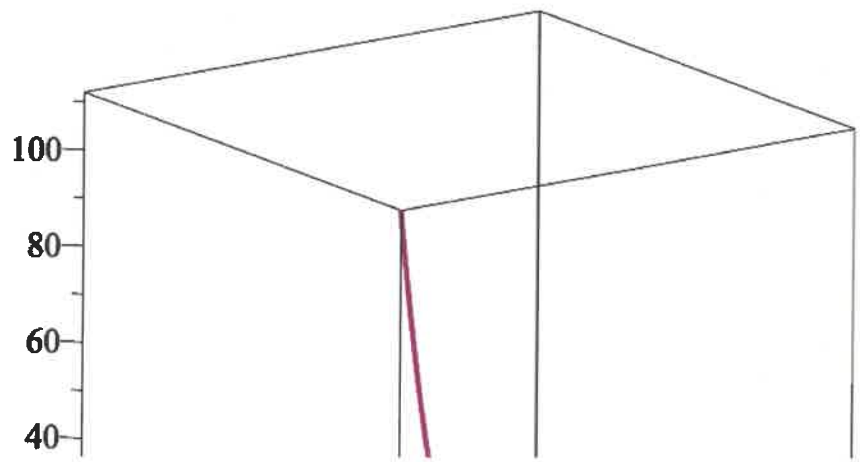
$xCurve := spacecurve([t, 3, f(t, 3)], t = -5..5], color = blue, thickness = 4)$



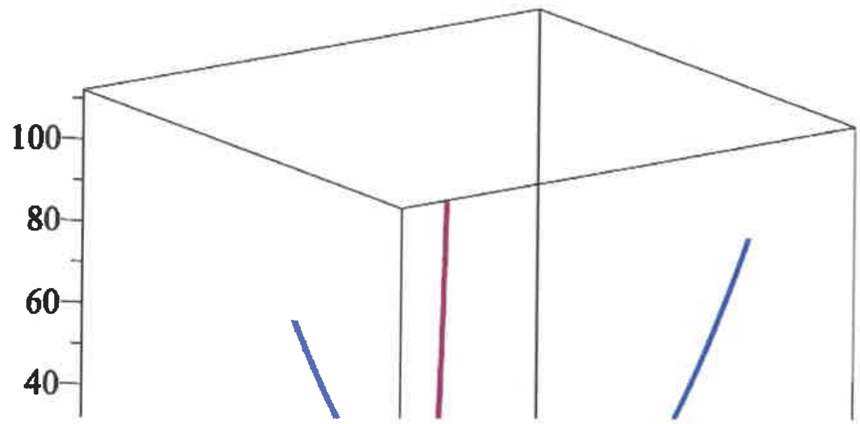
$yCurve := spacecurve([2, t, f(2, t), t = -5..5], color = red, thickness = 4)$



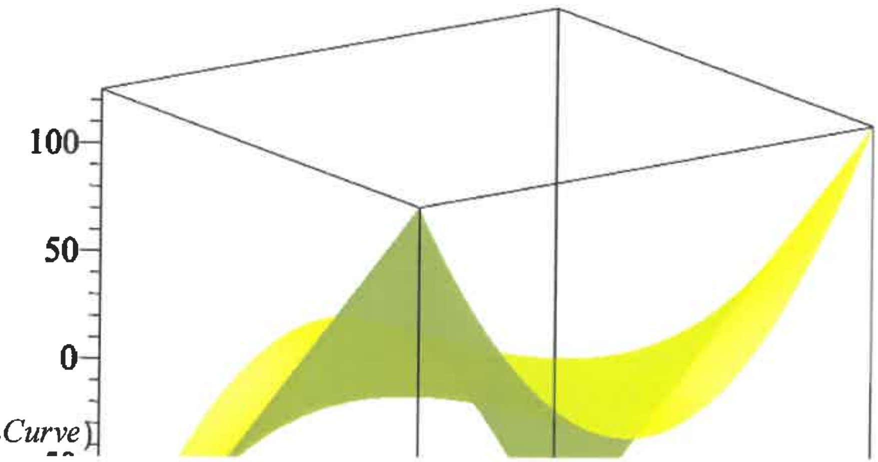
$LCurve := spacecurve([2 + t, 3 + 2\cdot t, f(t+2,\ 3+2\cdot t)], t = -2..2], color = purple, thickness = 4)$



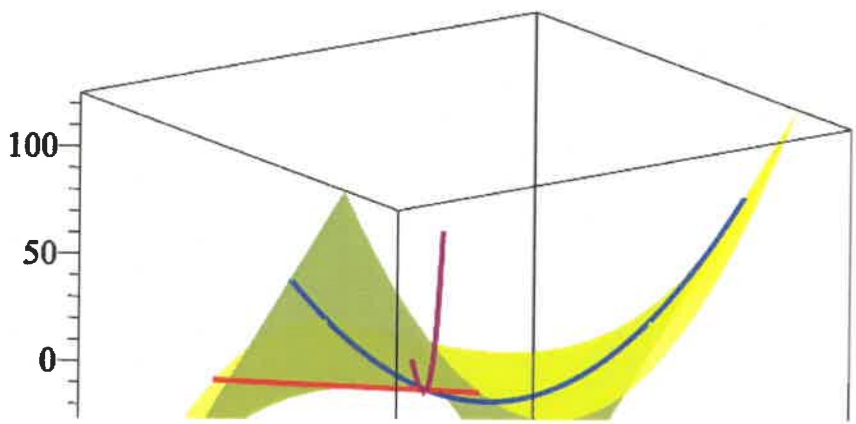
display(*xCurve*, *yCurve*, *LCurve*)



Surface := *plot3d*(*f* (*x*, *y*), *x* = −5 ..5, *y* = −5 ..5, *color* = *yellow*, *style* = *surface*, *transparency* = 0.3)



display(*Surface*, *xCurve*, *yCurve*, *LCurve*)



$f(x, y)$

$$x^2\, y \tag{2}$$

$fx := D[1](f)$

$$fx := (x, y) \mapsto 2\, y\, x \tag{3}$$

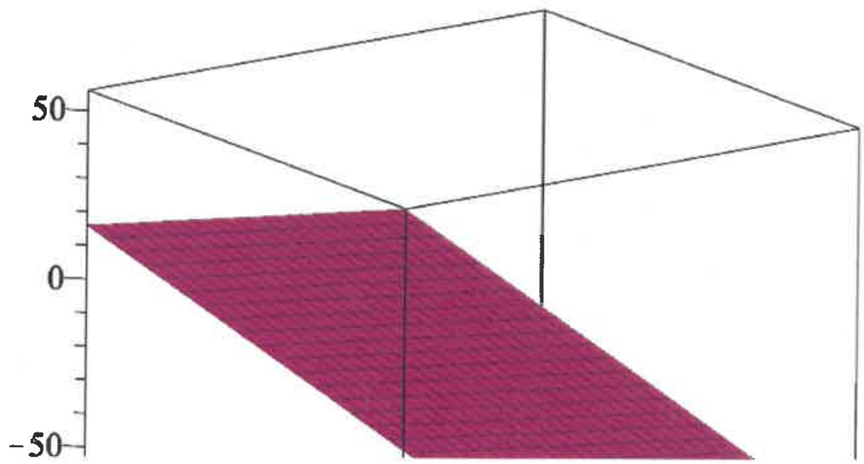
$fy := D[2](f)$

$$fy := (x, y) \mapsto x^2 \tag{4}$$

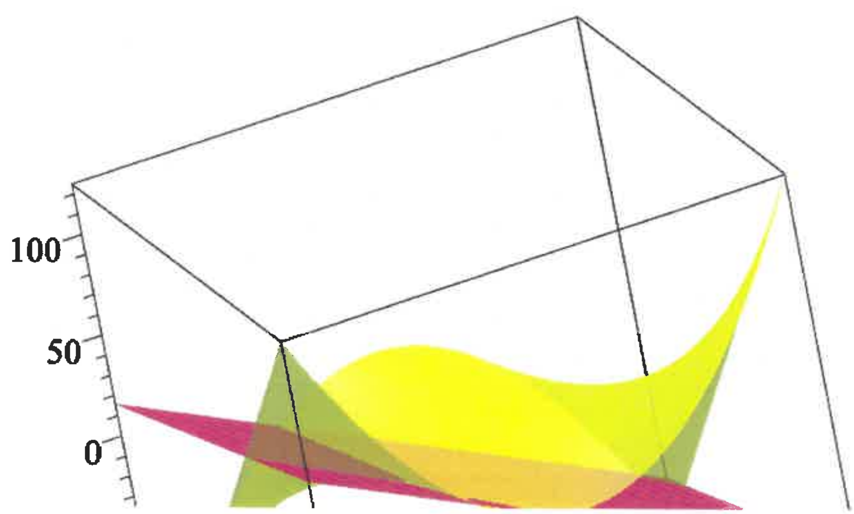
$T := (x, y) \to f(2, 3) + fx(2, 3) \cdot (x - 2) + fy(2, 3) \cdot (y - 3)$

$$T := (x, y) \mapsto f(2, 3) + fx(2, 3)\,(x - 2) + fy(2, 3)\,(y - 3) \tag{5}$$

$TangentPlane := plot3d(T(x, y), x = -5..5, y = -5..5, color = magenta)$

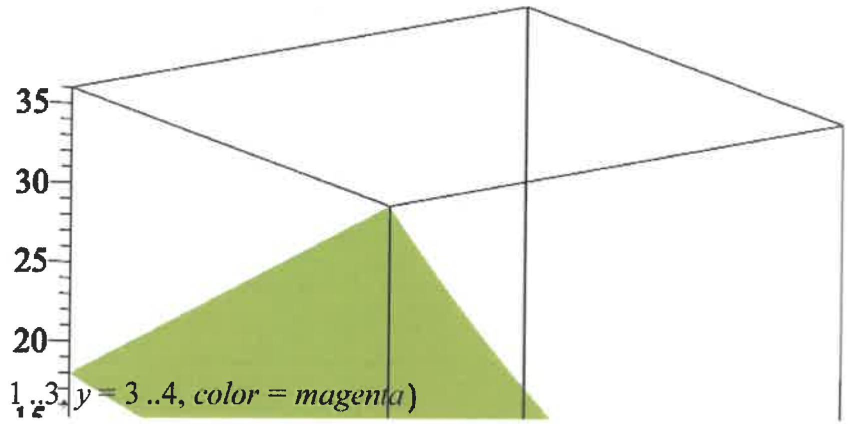


$display(Surface, TangentPlane)$

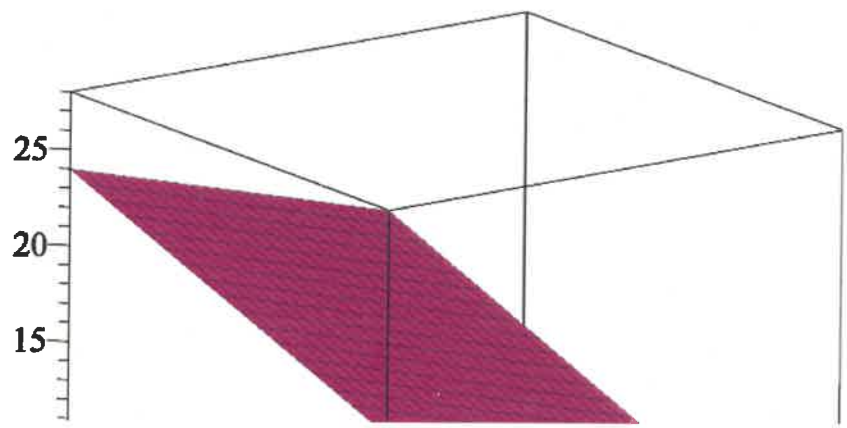


Zooming in:

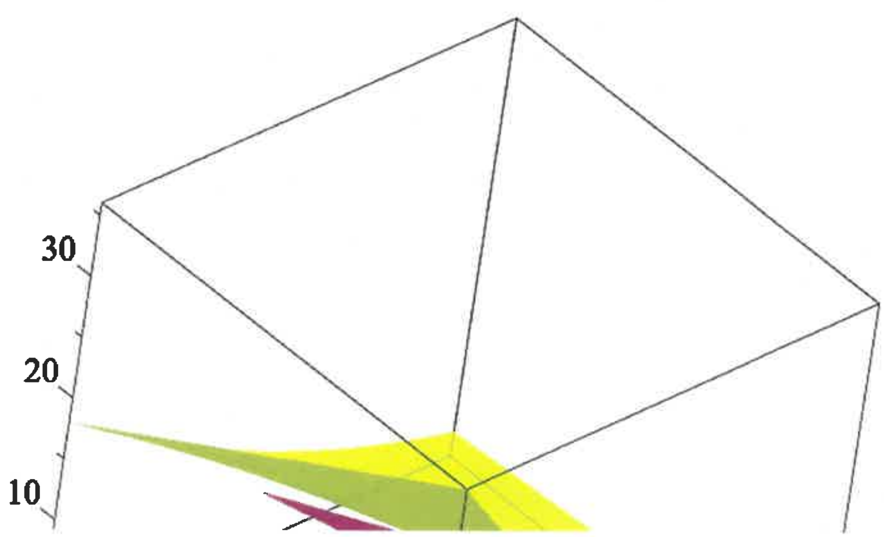
$Surface := plot3d(f(x, y), x = 1 ..3, y = 2 ..4, color = yellow, style = surface, transparency = 0.3)$



$TangentPlane := plot3d(T(x, y), x = 1 ..3, y = 3 ..4, color = magenta)$



$display(Surface, TangentPlane)$



Zooming in Further:

$Surface := plot3d(f(x, y), x = 1.5..2.5, y = 2.5..3.5, color = yellow, style = surface, transparency = 0.3):$

$TangentPlane := plot3d(T(x, y), x = 1.5..2.5, y = 2.5..3.5, color = magenta):$

$display(Surface, TangentPlane)$