CLASS 6

HANDOUTS

NOTES ON ASSIGNMENTS

Assignment 6

Announcements

EXAM 1: MONDAY, MARCH 7

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RE-EXAMINE $f(x,y) = \dfrac{x^2y}{x^2+2y^2}$ FOR $(x,y) \neq (0,0)$

$\lim_{(x,y)\to(0,0)} f(x,y)$ DOES NOT EXIST.

$\vec{x} = (x,y)$, $\vec{0} = (0,0)$

Let $\vec{x} \to \vec{0}$ along line $y = mx$

Then

$$f(x,y) = f(x,mx) = \frac{m}{1+2m^2}$$

m	$m/(1+2m^2)$
0	0
1	$1/3$
2	$2/9$
-1	$-1/3$

DERIVATIVE

$f: \mathbb{R}^1 \to \mathbb{R}^1 \qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\vec{f}: \mathbb{R}^1 \to \mathbb{R}^m \qquad \vec{f}'(x) = \lim_{h \to 0} \frac{\vec{f}(x+h) - \vec{f}(x)}{h}$

What about $f: \mathbb{R}^2 \to \mathbb{R}^1$

$$f'(\vec{x}) = \lim_{\vec{h} \to \vec{0}} \frac{f(\vec{x}+\vec{h}) - f(\vec{x})}{\vec{h}}$$

DIVISION by $\vec{h}$ makes no sense

Infinitely many ways $\vec{h} \to \vec{0}$

PARTIAL SOLUTION

CONSIDER 2 SPECIAL CASES for $\vec{h}$

① $\vec{h} = (t, 0)$

$$\lim_{t \to 0} \frac{f(x+t, y) - f(x,y)}{t}$$

$\partial f/\partial x, \; f_x, \; D[1](f)$

PARTIAL DERIVATIVE WITH Respect to x

TREAT y as a constant

Use usual rules of differentiation on x

② $h = (0, t)$

$$\lim_{t \to 0} \frac{f(x, y+t) - f(x,y)}{t}$$

$\frac{\partial f}{\partial y}, \; f_y, \; D[2](f)$

Partial Derivative with respect to y

Example $f(x,y) = x^2 y$

point $(3,4)$

$f_x(x,y) = 2xy$ $\qquad$ $f_y(x,y) = x^2$

$f_x(3,4) = 2 \cdot 3 \cdot 4 = 24$ $\qquad$ $f_y(3,4) = 9$

WORK OUT FROM DEFINITION

$$f_x(x,y) = \lim_{t \to 0} \frac{f(x+t,y) - f(x,y)}{t}$$

$$= \lim_{t \to 0} \frac{(x+t)^2 y - x^2 y}{t}$$

$$= \lim_{t \to 0} \frac{(x^2 + 2xt + t^2)y - x^2 y}{t}$$

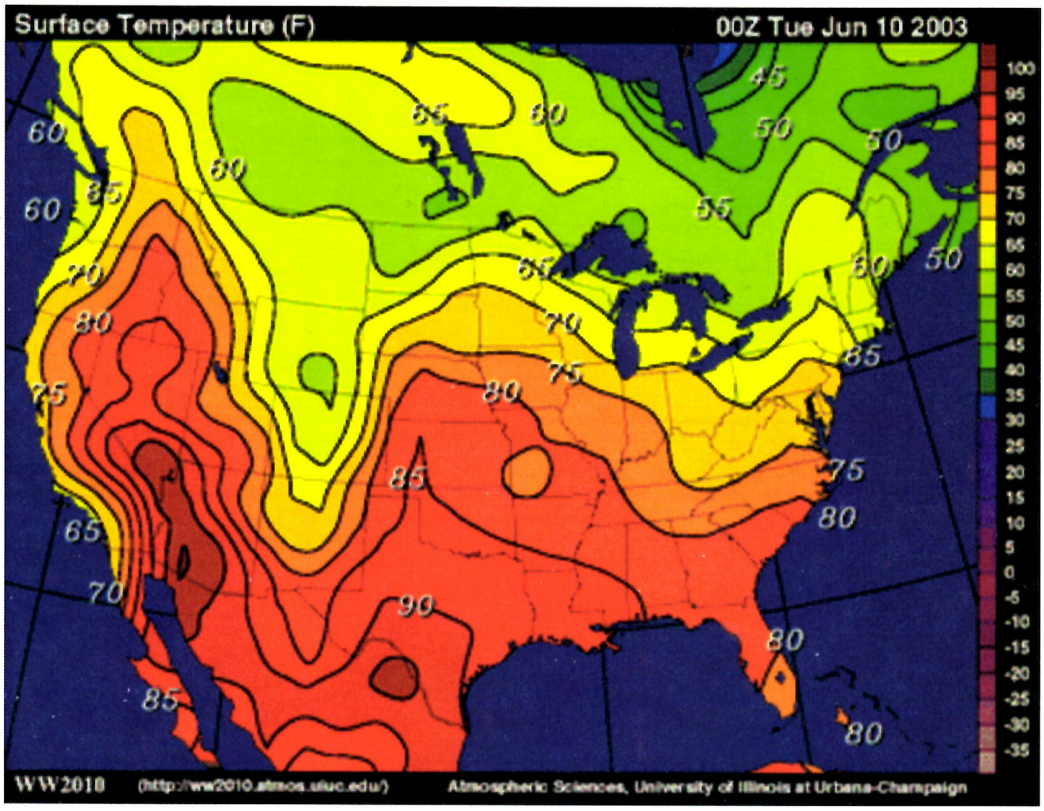
$$= \lim_{t \to 0} \frac{2xty + t^2 y}{t}$$

$$= \lim_{t \to 0} (2xy + ty)$$

$$= 2xy$$

What is geometric meaning?

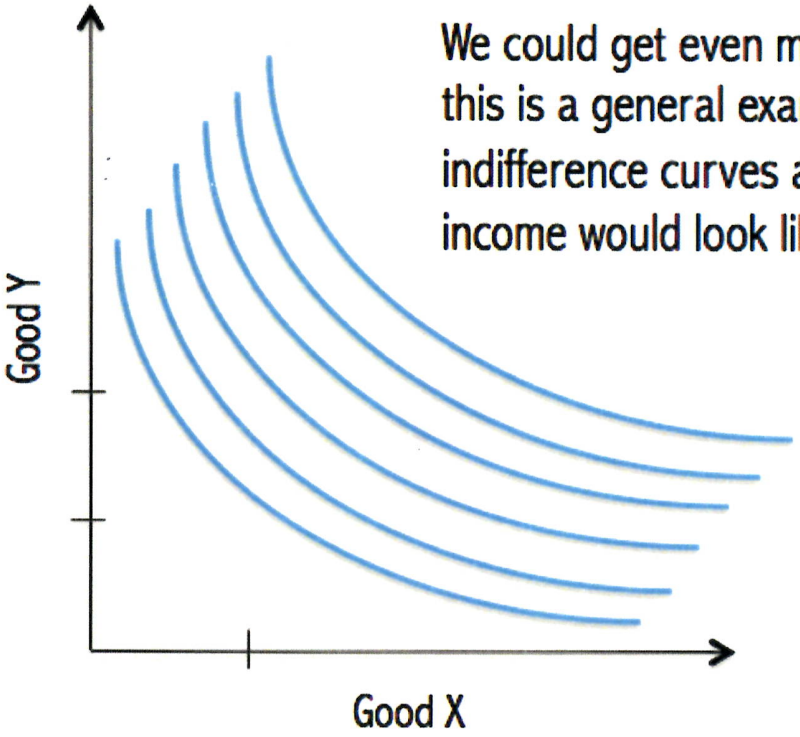
Other Instances of Level Curves



Today's Weather

Topographic Curve

Indifference Curves



We could get even more detailed, but this is a general example of what indifference curves at different levels of income would look like for one person.