CLASS 5

HANDOUTS

NOTES ON ASSIGNMENT 4

ASSIGNMENT 5

TODAY: BEGIN EXAMINATION of $f: \mathbb{R}^n \longrightarrow \mathbb{R}^1$.

REAL-VALUED FUNCTIONS of VECTORS

(I) Defining and using DERIVATIVE

Chapters 3 - 5

(II) INTEGRAL

CHAPTERS 6-7

INITIAL FOCUS: GEOMETRY OF SUCH FUNCTIONS

USEFUL PICTURES

(A) $f: \mathbb{R}^2 \to \mathbb{R}^1$ $\qquad z = f(x, y)$

IMAGE : INTERVAL OF REAL NUMBERS

DOMAIN: REGION IN PLANE

GRAPH: $\boxed{\text{SURFACE}}$ IN $\mathbb{R}^3$

IN GENERAL, graph of $f: \mathbb{R}^n \longrightarrow \mathbb{R}^1$ is the set of points of form $(\vec{x}, f(\vec{x}))$, an n-dimensional SURface IN $(n+1)$-dimensional SPACE.

(B) CONTOURS: Level Curves for $f: \mathbb{R}^2 \to \mathbb{R}^1$

Level Surfaces: $f: \mathbb{R}^3 \to \mathbb{R}^1$

(C) CROSS-SECTIONS

FIX one Variable

<u>Example</u>: $f(x, y) = 2x^2 + 3y^2$

OR $z = 2x^2 + 3y^2$

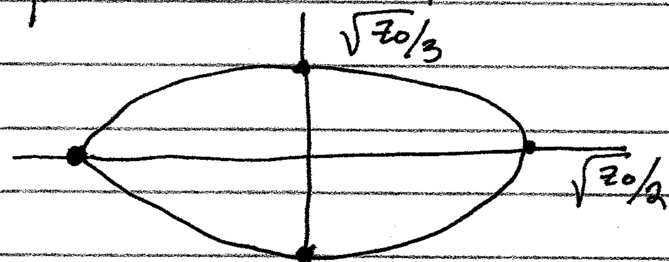
OBSERVATIONS

① $z \geq 0$ for all x, y and $z = 0$ only at $(0, 0)$

② Hold z constant, say $z = z_0 > 0$

$$2x^2 + 3y^2 = z_0$$

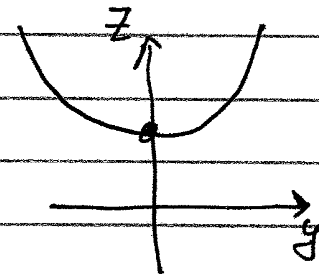
equation in (x, y) plane



③ HOLD x- constant; $x = x_0$

$$2x_0 + 3y^2 = z$$

PARABOLA IN (y, z) plane



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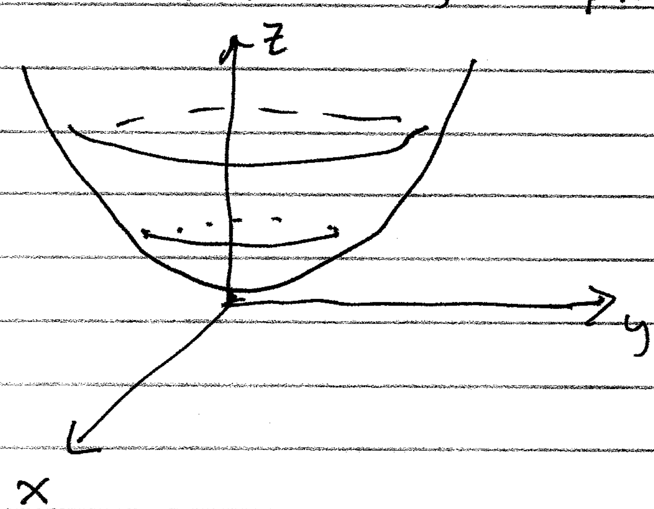


Level Curves

GIVEN: output k (some constant)

FIND: All inputs which produce that output

Examples

ISOTHERMS

ISOBARS

ISOCLINES

INDIFFERENCE CURVES

See Maple WORKSHEET

FUNDAMENTAL DIFFERENCE BETWEEN DOMAIN BEING subset of $\mathbb{R}^1$ and subset of $\mathbb{R}^n$, $n>1$:

IMPLICATION FOR

CONTINUITY

DERIVATIVE

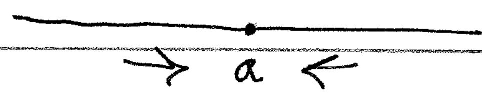
INTEGRAL

THESE ALL DEPEND ON LIMIT

$$\lim_{x \to a} f(x)$$

$\mathbb{R}^1$

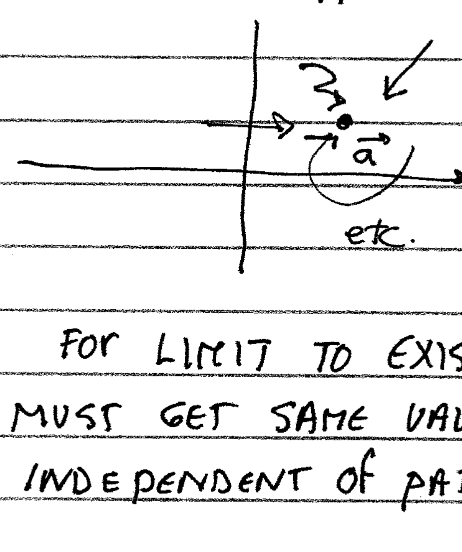
2 ways to approach a



$$\lim_{\vec{x} \to \vec{a}} f(\vec{x})$$

$\mathbb{R}^n$

infinitely many ways for $\vec{x}$ to approach $\vec{a}$

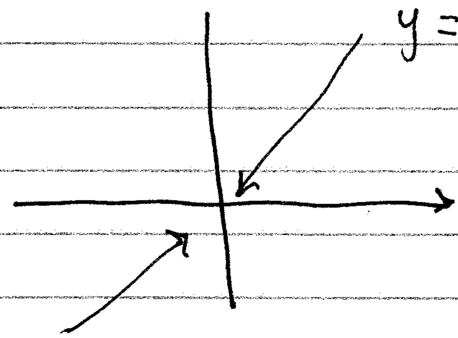


FOR LIMIT TO EXIST, MUST GET SAME VALUE INDEPENDENT OF PATH.

Example

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+2y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

APPROACH ALONG LINE $y = mx$



THEN $f(x, mx) = \dfrac{x(mx)}{x^2+2(mx)^2} = \dfrac{m x^2}{x^2+2m^2x^2}$

$$= \frac{m x^2}{x^2(1+2m^2)} = \frac{m}{1+2m^2}$$

$m = 1$: $\quad \frac{1}{1+2} = 1/3$

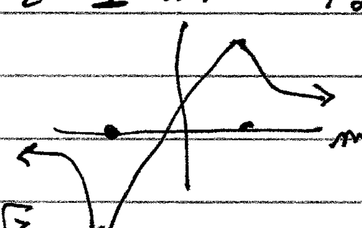
$m = 2$: $\quad \frac{2}{1+8} = 2/9$

$m = -1$: $\quad -1/3 = -1/3$

What possible VALUES CAN $g(m) = \dfrac{m}{1+2m^2}$?

NOTE $g'(m) = \dfrac{1-2m^2}{(1+2m^2)^2}$ $\quad$ POSITIVE FOR $-\sqrt{2}/2 \le m \le \sqrt{2}/2$

$g''(m) = \dfrac{4m(2m^2-3)}{(1+m^2)^3}$



$g(m) \to 0$ as $m \to \pm\infty$

pt of inflection at $m = \pm\sqrt{3}$

Range $\cancel{-1/3 \quad 1/3}$ $\quad \left[-\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}\right]$

$$\frac{m}{1+2m^2} = k$$

$$k + 2km^2 = m$$
$$2km^2 - m + k = 0$$

$$m = \frac{1 \pm \sqrt{1-8k^2}}{4k}$$

$$1 - 8k^2 \geq 0$$
$$1 \geq 8k^2$$
$$k^2 \leq 1/8$$
$$|k| < 1/\sqrt{8} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$