CLASS 11

HANDOUTS

NOTES ON EXAM 1

NOTES ON ASSIGNMENT 10

ASSIGNMENT 11

EXAM 1: Median Grade 95

LIMITS and CONTINUITY

r-neighborhood of $\vec{a}$ IN $\mathbb{R}^n$ is all vectors $\vec{x}$ such that $|\vec{x}-\vec{a}| < r$ where $r > 0$

INTERIOR POINT

BOUNDARY POINT

LIMIT POINT

Open Sets

CLOSED Set

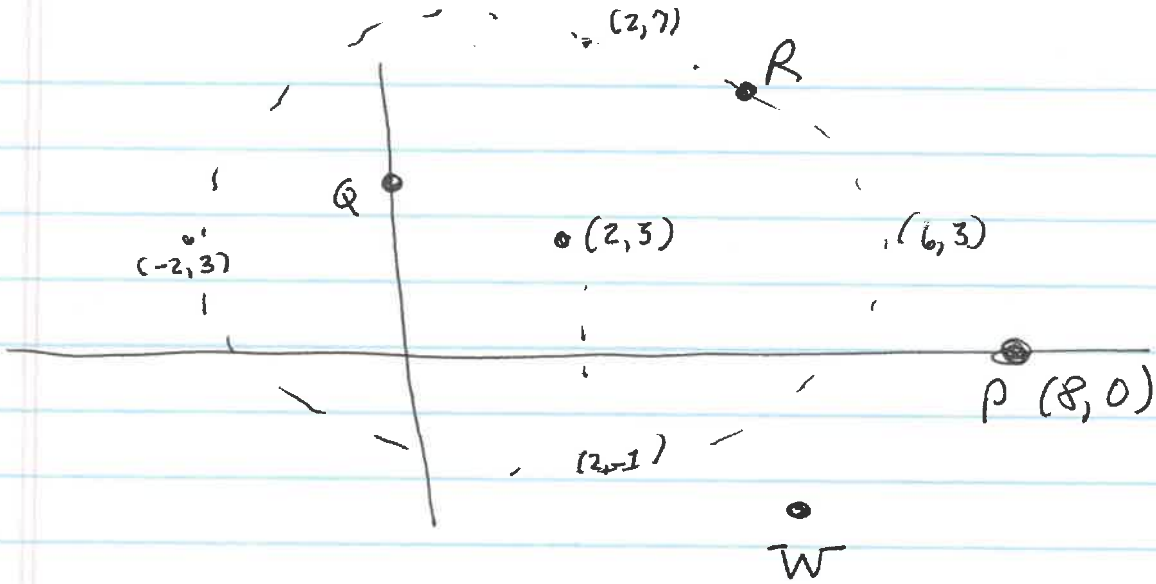
Example Let $S = \{ \vec{x} : |\vec{x} - (2,3)| < 4 \} \cup \{ (8,0) \}$

POINT	INTERIOR POINT	LIMIT POINT	Boundary Point
Q	YES	YES	NO
R	NO	YES	YES
P	NO	NO	YES
W	NO	NO	NO

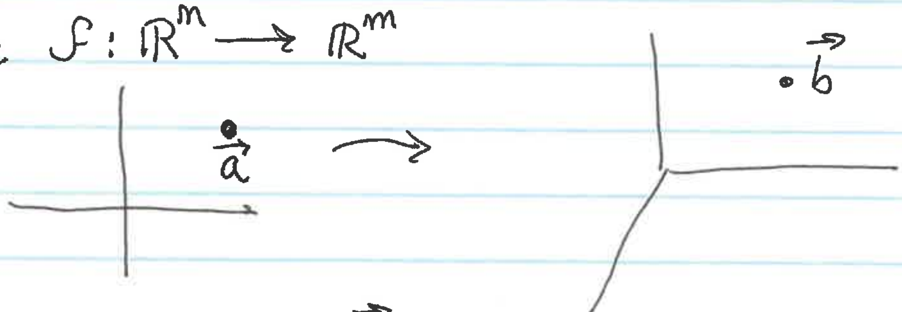
$\sin x = 0 \quad x = 0, \pi, 2\pi, 3\pi, \ldots$

$\sin x = 1 \quad x = \pi/2, \frac{\pi}{2} + 2\pi = \frac{5\pi}{2}, 9\pi/2$

$\sin x = -1 \quad x = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \frac{15\pi}{2}, \ldots$



Let $f: \mathbb{R}^n \longrightarrow \mathbb{R}^m$



$$\lim_{\vec{x} \to \vec{a}} f(\vec{a}) = \vec{b}$$

$$\Updownarrow$$

MEANS for every ε-nbhd of $\vec{b}$, there is a δ-nbhd U of $\vec{a}$ such that $\vec{x}$ in U $(\vec{x} \neq \vec{a})$, then $f(\vec{x})$ is in V

f is continuous at $\vec{a}$ if there is a $\vec{b}$ such that

$$\lim_{\vec{x} \to \vec{a}} f(\vec{x}) = \vec{b}$$

and $f(\vec{a}) = \vec{b}$

$f: \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at $\vec{a}$
if there is an $m \times n$ matrix M
such that

$$\lim_{\vec{x} \to \vec{a}} \frac{\overrightarrow{f(\vec{x}) - f(\vec{a}) - M(\vec{x}-\vec{a})}}{|\vec{x}-\vec{a}|} = \vec{O}$$

BIG THEOREMS

If f is differentiable at $\vec{a}$, then f is continuous at $\vec{a}$

If all PARTIAL DERIVATIVES of $\vec{f}$ are CONTINUOUS IN a neighborhood of $\vec{a}$, then $\vec{f}$ is differentiable at $\vec{a}$

If f is diff at $\vec{a}$, then M is the matrix of first order partial derivatives

SPECIAL CASE: $f: \mathbb{R}^n \to \mathbb{R}^1$
Then M is the $1 \times n$ matrix of PARTIALS $= \nabla f(\vec{a})$

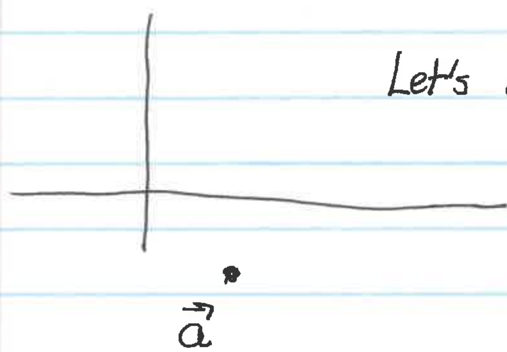
DIRECTIONAL DERIVATIVE

Let $f(x,y) = 3x^2 + 5xy - 2y^2 + 1$

$\vec{a} = (2,-1)$

Then $f(\vec{a}) = f(2,-1) = +1$

Let's approach $\vec{a}$ along $\vec{v} = (v_1, v_2)$



PARTIAL DERIVATIVE WITH RESPECT TO $\vec{V}$ at $\vec{a}$ is

$$\lim_{t \to 0} \frac{f(2+tV_1, -1+tV_2) - f(2,1)}{t}$$

$$\lim_{t \to 0} \frac{3(2+tV_1)^2 + 5(2+tV_1)(-1+tV_2) - 2(-1+tV_2)^2 + 1 - \{3 \cdot 2^2 + 5(2)(-1) - 2(-1)^2 + 1)}{t}$$

$$3\left[2^2 + 4tV_1 + t^2V_1^2\right] + 5\left((2)(-1) + 2tV_2 - tV_1 + t^2V_1V_2\right) - 2\left[(-1)^2 + -2tV_2 + t^2V_2^2\right] + 1$$

$$- (\qquad \qquad)$$

Let $f(x,y) = x^2 y$

$\vec{a} = (3,9)$

$f(\vec{a}) = f(3,9) = 3^2 \cdot 9 = 81$

Approach $(3,9)$ along $\vec{V}$ ~~(v,w)~~) $\vec{V} = (V_1, V_2)$

PARTIAL DERIVATIVE of f with respect to $\vec{V}$ at $\vec{a}$

$$f_{V,(\omega)}(\vec{a}) = \lim_{t \to 0} \frac{f(\vec{a} + t\vec{V}) - f(\vec{a})}{t}$$

$$= \lim_{t \to 0} \frac{f(3 + tV_1, 9 + tV_2) - f(3,9)}{t}$$

$$= \lim_{t \to 0} \frac{(3+tV_1)^2 (9+tV_2) - (3^2)(9)}{t}$$

$$= \lim_{t \to 0} \frac{(3^2 + 6tV_1 + t^2V_1^2)(9+tV_2) - (3^2)(9)}{t}$$

$$= \lim_{t \to 0} \frac{3^2 \cdot 9 + 3^2 \cdot tV_2 + 6tV_1 \cdot 9 + 6t^2 V_1 V_2 + t^2 V_1^2 9 + t^3 V_1^2 V_2 - (3^2)(9)}{t}$$

$$= \lim_{t \to 0} 3^2 V_2 + 54 V_1 + 6tV_1V_2 + tV_1^2 9 + t^2 V_1^2 V_2$$

$$= 3^2 V_2 + 54 V_1 + 6 V_1 V_2$$