CLASS 3

HANDOUTS

NOTES ON ASSIGNMENT 2

ASSIGNMENT 3

CIRCLES AND ELLIPSES (MAPLE) IN HANDOUT FOLDER

DERIVATIVES AND INTEGRALS FOR

$$\vec{F} : \mathbb{R}^1 \longrightarrow \mathbb{R}^m$$

LOOK AT $\vec{F} : \mathbb{R}^1 \longrightarrow \mathbb{R}^2$

$$\vec{F}(x) = (f(x), g(x))$$

DIFFERENCE QUOTIENT

$$\frac{\vec{F}(x+h) - \vec{F}(x)}{h} = \left(\frac{f(x+h) - f(x)}{h}, \frac{g(x+h) - g(x)}{h} \right)$$

so

$$\vec{F}'(x) = \lim_{h \to 0} \frac{\vec{F}(x+h) - \vec{F}(x)}{h} = (f'(x), g'(x))$$

Example $\vec{F}(x) = (\cos x, x^3 - 2x)$

$\Rightarrow \vec{F}'(x) = (-\sin x, 3x^2 - 2)$

Example $\vec{F}(x) = (\tan t, \ln t)$

$\Rightarrow \vec{F}'(x) = (\sec^2 t, 1/t)$

NOTHING SPECIAL ABOUT $m = 2$

$$\vec{F}(t) = (f_1(t), f_2(t), \ldots, f_m(t))$$

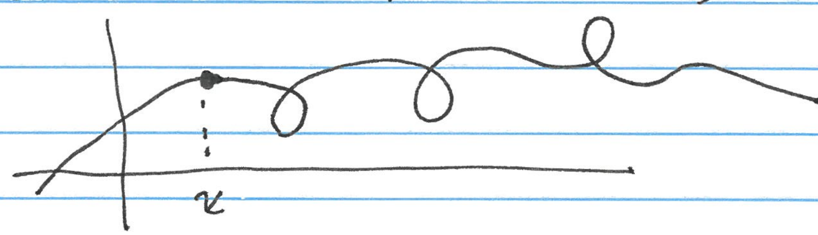
THEN

$$\vec{F}'(t) = (f_1'(t), f_2'(t), \ldots, f_m'(t))$$

Example $\vec{F}(t) = (t^7, t^{-3}, \sin(t^2))$

$$\Rightarrow \vec{F}'(t) = (7t^6, -3t^{-4}, 2t\cos(t^2))$$

Image of $\vec{F}$ IS CURVE (1 dimensional) IN $\mathbb{R}^m$



Tangent Line $L(t) = \vec{F}(x) + t\vec{F}'(x)$

Example $\vec{F}(x) = (x^3 + 7x + 3, 8 + \sin x)$

so $\vec{F}'(x) = (3x^2 + 7, \cos x)$

at $x = 0$

$$\vec{F}(0) = (3, 8)$$

$$\vec{F}'(0) = (7, 1)$$

so tangent line equation is

$$\vec{L}(t) = (3, 8) + t(7, 1)$$

$$\vec{L}(t) = (3 + 7t, 8 + t)$$

we can write as $x = 3 + 7t$, $y = 8 + t$

so $t = \frac{x-3}{7}$ and $y = 8 + \frac{x-3}{7}$

OR $y = \frac{x}{7} + 8 - \frac{3}{7} = \frac{x}{7} + \frac{53}{7}$

Bottom Line

$$\vec{F} = (f_1, f_2, \ldots, f_m)$$

$\vec{F}$ is continuous $\Leftrightarrow$ all f_i are continuous

$\vec{F}$ is differentiable $\Leftrightarrow$ all f_i are differentiable

and

$$\vec{F}' = (f_1', f_2', \ldots, f_m')$$

and

$$\int \vec{F} = \left(\int f_1, \int f_2, \ldots, \int f_m \right)$$

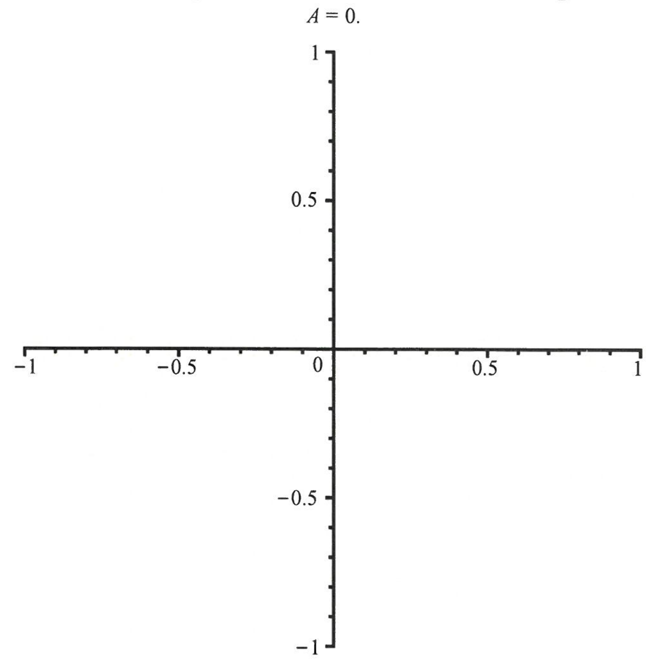
KEY STEP IS THEOREM 2.2.1

Parametrization of Circles and Ellipses in the Plane

Standard Unit Circle $x^2 + y^2 = 1$

$with(plots):$

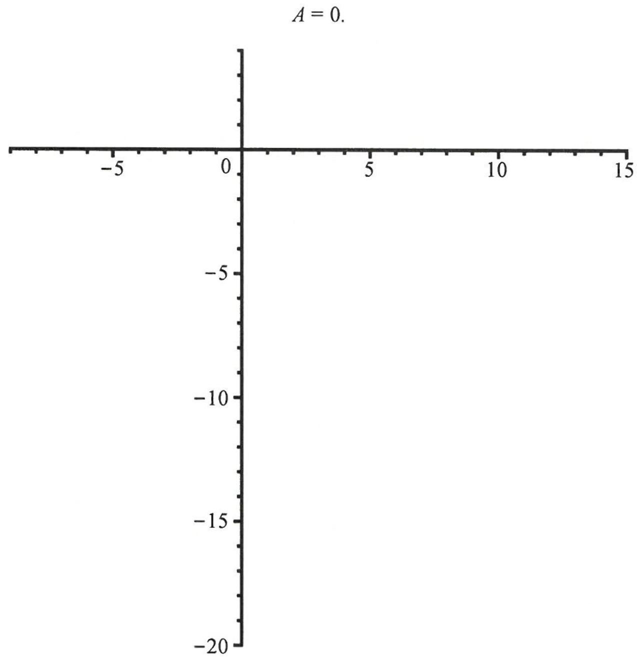
$animate(\ plot,\ [\ [\cos(t), \sin(t), t = 0..A], color = blue, thickness = 4], A = 0..2\cdot\pi)$



Circle of radius 12 with center at (3, -8)

$$(x - 3)^2 + (y + 8)^2 = 144$$

$animate\left(plot, [[3 + 12 \cdot \cos(t), -8 + 12 \cdot \sin(t), t = 0 ... A], color = magenta, thickness = 4], A = 0 ..2\cdot\pi\right)$

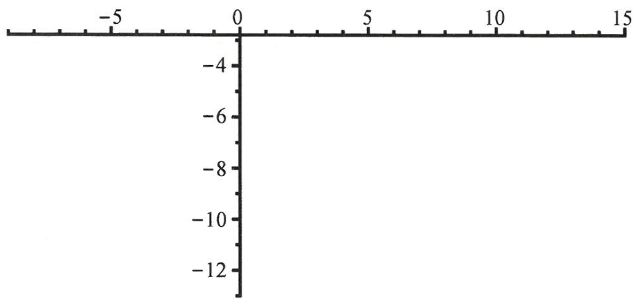


An Ellipse with Center at (-3,8)

$$\frac{(x-3)^2}{12^2} + \frac{(y+8)^2}{5^2} = 1$$

$animate\left(plot, [[3 + 12 \cdot \cos(t), -8 + 5 \cdot \sin(t), t = 0 ... A], color = green, thickness = 4, scaling = constrained], A = 0..2 \cdot \pi \right)$

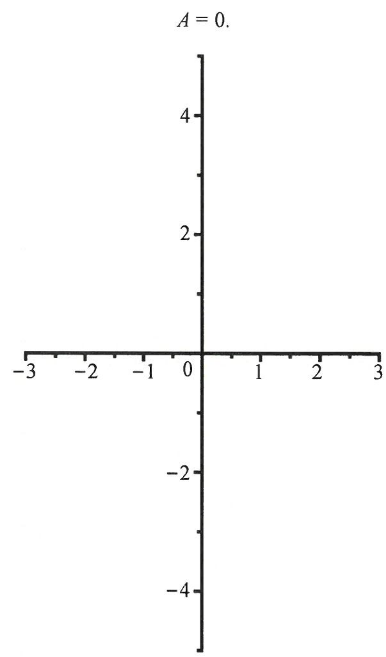
$A = 0.$



An Ellipse Centered at the Origin

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

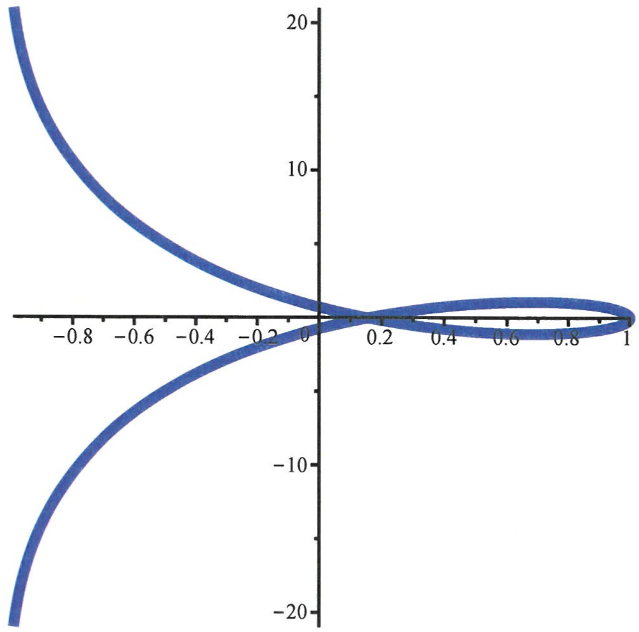
$animate\left(plot, [[3 \cdot \cos(t), 5 \cdot \sin(t), t = 0 ...A], color = blue, thickness = 4, scaling = constrained], A = 0 ..2 \cdot \pi \right)$



Some Class 3 Examples

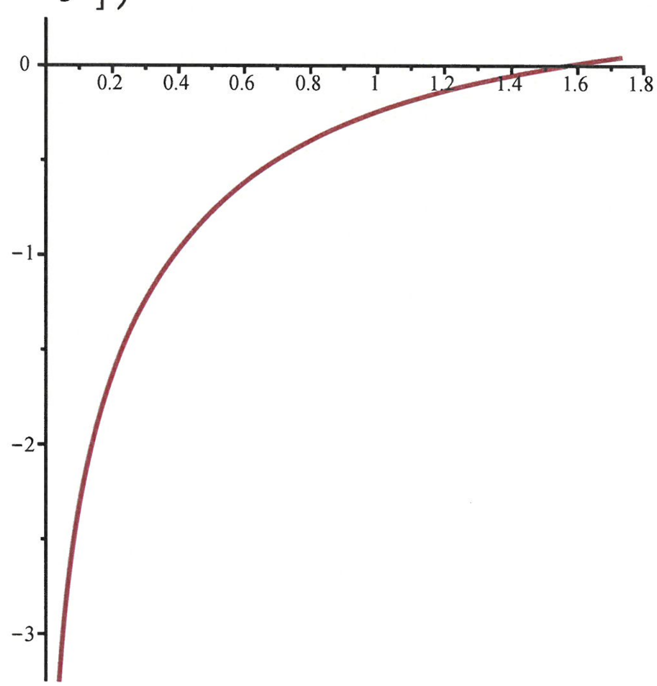
Plotting $F(x) = (\cos x, x^3 - 2x)$

$plot([\cos(x), x^3 - 2x, x = -3..3], color = blue, thickness = 4)$



Plotting $G(x) = (\tan t, \ln t)$

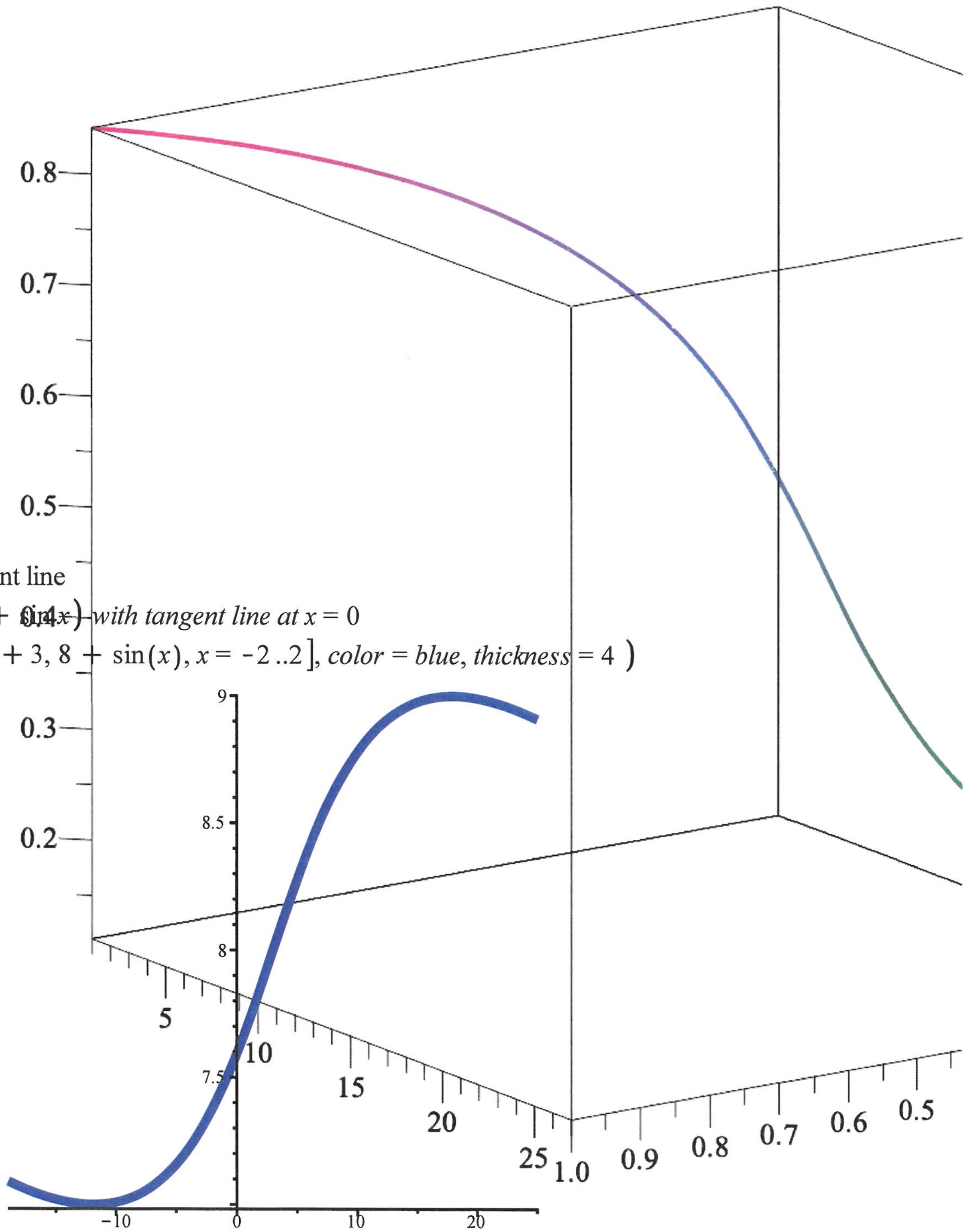
$plot\left(\left[\tan(t), \log(t), t = 0..\frac{Pi}{3}\right]\right)$



Plotting a Curve in 3-space:

$with(plots):$

$spacecurve\left([t^7, t^{-3}, \sin(t^2)], t = \frac{1}{3}..1, thickness = 4\right)$

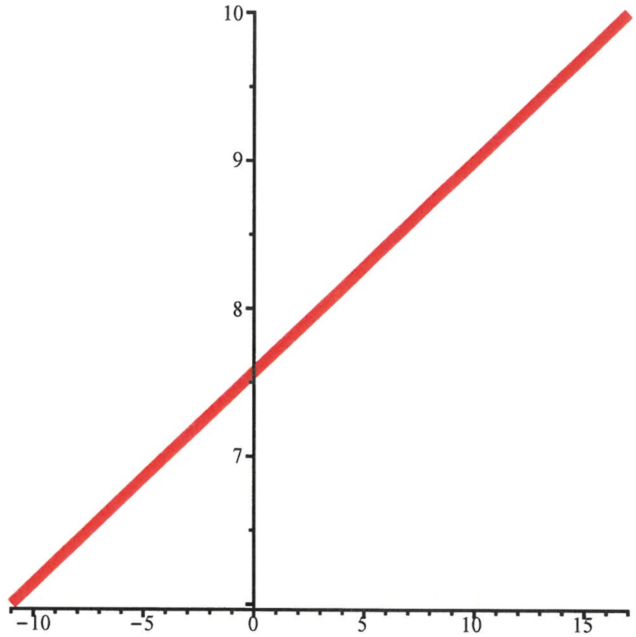


Example of curve and tangent line

$F(x) = (x^3 + 7x + 3, 8 + \sin x)$ *with tangent line at* $x = 0$

$Curve := plot(\,[x^3 + 7\cdot x + 3, 8 + \sin(x), x = -2..2], color = blue, thickness = 4\,)$

$Tangent := plot([3 + 7\cdot t, 8 + t, t = -2..2], color = red, thickness = 4)$



display(*Curve*, *Tangent*)