MATH 223 MULTIVARIABLE CALCULUS CLASS 1

MONDAY 2/14/22

HANDOUTS

① COURSE PACKET

Course Description
Topics
SCHEDULE
READING MATH BOOKS
STUDYING AND LEARNING
ON PROBLEM SOLVING
Vectors IN $\mathbb{R}^2$ and $\mathbb{R}^3$
ASSIGNMENT 1

② QUESTIONNAIRE

HIGHLIGHTS FROM COURSE PACKET

WE ASSUME YOU HAVE READ TEXT CAREFULLY AND FULLY

CLASSTIME

ANSWERS YOUR QUESTIONS
PROVIDE BIG PICTURE
LOOK IN DETAIL AT SOME INTERESTING APPLICATIONS AND PROBLEMS
DO SOME PROBLEMS IN SMALL GROUPS

DERIVATION OF KEPLER'S LAWS
ISAAC NEWTON (1643 - 1727)

THIS IS A CALCULUS COURSE

$\Rightarrow$ WE DEAL WITH POWERFUL IDEAS:

LIMITS, DERIVATIVES, INTEGRALS

OF FUNCTIONS

CLASSIC SETTING: $y = f(x)$ $\quad f: \mathbb{R}^1 \to \mathbb{R}^2$

input and output are SINGLE NUMBERS

GRAPH IS A CURVE (1 dimensional) IN PLANE ($\mathbb{R}^2$)

IDEA OF FUNCTION GENERALIZES EASILY

input: ONE OBJECT

output: ONE OBJECT

FOR US: OBJECTS ARE VECTORS

VECTOR IS UNIFYING CONCEPT OF

LINEAR ALGEBRA AND MULTIVARIABLE CALCULUS

CALCULUS I AND II: REAL-VALUED FUNCTION OF REAL VARIABLE

MULTIVARIABLE: VECTOR-VALUED FUNCTION OF VECTOR VARIABLE

ULTIMATE GOAL: $f: \mathbb{R}^n \to \mathbb{R}^m$

EXAMPLE Let A be 3×4 matrix
$\vec{x}$ a 4×1 vector.

$$\begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix} \begin{bmatrix} - \\ - \\ - \\ - \end{bmatrix} = \begin{bmatrix} - \\ - \\ - \end{bmatrix}$$

$$\underset{A}{3 \times 4} \qquad \underset{\vec{x}}{4 \times 1} \qquad \underset{\vec{b} = A\vec{x}}{3 \times 1}$$

A gives a function from $\mathbb{R}^4$ to $\mathbb{R}^3$

CLASSIC APPLICATIONS OF CALCULUS

- Motion of object moving along STRAIGHT LINE
  POSITION, VELOCITY, ACCELERATION
- PROFIT AS FUNCTION OF PRICE
- AMOUNT OF DRUG IN BLOODSTREAM AT TIME t

REAL WORLD IS MUCH MORE COMPLICATED

- MOTION: objects move in plane or space
  Need vector to describe location
- PROFIT: depends on prices, demand, taxes, production costs
- GPA: function of many course grades

INPUT: COVID BUDGET

OUTPUT: How much to spend on masks, vaccines, PPE, etc.

DEFINITION OF DERIVATIVE

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$g'(x) = \lim_{k \to 0} \frac{g(x+k) - g(x)}{k}$$

IF THE LIMIT EXISTS

<u>Example</u> FIND $f'(x)$ IF $f(x) = x^3$

<u>SOLUTION</u> $f(x+h) - f(x) = (x+h)^3 - x^3$

$= x^3 + 3x^2h + 3xh^2 + h^3 - x^3$

$= 3x^2h + 3xh^2 + h^3$

$= h(3x^2 + 3xh + h^2)$

so

$$\frac{f(x+h) - f(x)}{h} = 3x^2 + 3xh + h^2 \quad \text{if } h \neq 0$$

$$f'(x) = \lim_{h \to 0} \left[3x^2 + 3xh + h^2\right] = 3x^2$$

Example Determine $g'(x)$ if $g(x) = x\,f(x)$
and f is a differentiable function

Solution

$$g(x+h) - g(x) = (x+h)\,f(x+h) - x\,f(x)$$
$$= x\,f(x+h) + h\,f(x+h) - x\,f(x)$$
$$= x\,[f(x+h) - f(x)] + h\,(f(x+h))$$

So

$$\frac{g(x+h) - g(x)}{h} = x\,\frac{f(x+h) - f(x)}{h} + f(x+h).$$

Taking limit as $h \to 0$

$$\frac{f(x+h) - f(x)}{h} \to f'(x)$$

$$f(x+h) \to f(x) \quad [f \text{ is continuous}]$$

Hence $g'(x) = x\,f'(x) + f(x)$