MATH 223

Some Hints and Answers for Assignment 33

Exercises 36ac, 38, and 39 of Chapter 8.

36: Sketch a picture of each of the regions $\mathcal{R}$ in $\mathbb{R}^3$ described below along with a representative number of outward-pointing normals.Then verify the correctness of Gauss's Theorem for the given vector fields.

Notes: One region is a solid ball and the other is a solid circular cylinder.
For (a): The Solution to Exercise 27 of a previous assignment will be helpful. Show $\int_{\partial\mathcal{R}} \mathbf{F} = \int_{\partial\mathcal{R}} 0 = 0$.

For (c):

(c) div $\mathbf{F} = 0_x + y_y + 0_z = 1$ so $\int_{\mathcal{R}}$ div $\mathbf{F} = \int_{\mathcal{R}} 1 =$ Volume enclosed by $\mathcal{R}$ but this is the volume of a circular cylinder of radius 2 and height 1 which has volume 4π.

The base of the cylinder lies in the xy-plane. The surface has three components:

The Top T:	The Bottom B:	The Cylindrical Side C: $(x^2 + y^2 = 4, 1 \le z \le 2)$
$(x^2 + y^2 \le 4z = 2)$	$(x^2 + y^2 \le 4, z = 1)$	$(x^2 + y^2 = 4, 1 \le z \le 2)$

We can parametrize them as follows:
Top T: $f(s, t) = (2s \cos t, 2s \sin t, 2), 0 \le s \le 1, 0 \le t \le 2\pi$
Side C : $g(s, t) = (2 \cos s, 2 \sin s, t), 0 \le s \le 2\pi, 0 \le t \le 1)$
Bottom B: $h(s, t) = (2t \cos s, 2t \sin s, 1), 0 \le s \le 2\pi, 0 \le t \le 2\pi$

38: The flux is 3 times the volume enclosed by the upper hemisphere (see Exercise 39);

39: Show divergence of the vector field is 3 and then use Gauss's Theorem.