MATH 223
Hints and Answers for Assignment 32
Exercises 27, 29a and 31 of Chapter 8.

27: If S is a sphere, what is its radius and where is its center?
$\sigma_s(s,t) \times \sigma_t(s,t) = \big(-a^2 \cos s \sin^2 t, -a^2 \sin s \sin^2 t, -a^2 \sin t \cos t\big)$ and $|\sigma_s(s,t) \times \sigma_t(s,t)| = a^2 |\sin t|$..
Note that $|\sin t|$ is $\sin t$ for $0 \le t \le \pi$ but $|\sin t| = -\sin t$ for $\pi \le t \le 2\pi$. You can exploit the symmetry of the sphere to calculate the surface area of one hemisphere and double the answer.
The Area$(S) = 4a^2\pi$.

29a: $$\iint_S f \, d\sigma = \int_{t=0}^{t=1} \int_{s=0}^{s=1} s + t\sqrt{4t^2+1} \, ds \, dt = \int_{t=0}^{t=1} \frac{1}{2} + t\sqrt{4t^2+1} \, dt = \frac{5}{12}(1+\sqrt{5})$$

31: There is a typo in the text. The vector field should be $\mathbf{F}(x,y,z) = (x, 2y, 3z)$. We have $\int_S \mathbf{F} \cdot d\mathbf{S} = \int_{t=0}^{t=1} \int_{s=0}^{s=1} 12st - 2s^2 - t^2 \, ds \, dt = \int_{t=0}^{t=1} -\frac{2}{3} + 6t - t^2 \, dt = 2$.