1. Formally encoding a relation \( R \) on a set \( X \) as a subset \( R \subseteq X \times X \):

(a) "A R B" becomes \((a, b) \in R\).

(b) "\( R \) is reflexive" becomes \( \forall a \in X, (a, a) \in R \).

(c) "\( R \) is symmetric" becomes \( \forall a, b \in X, (a, b) \in R \Rightarrow (b, a) \in R \).

(d) "\( R \) is transitive" becomes \( \forall a, b, c \in X, (a, b) \in R \land (b, c) \in R \Rightarrow (a, c) \in R \).

(e) "\( S \) subsumes \( R \)" becomes \( S \supseteq R \).

2. Encoding a relation from \( A \) to \( B \) as a subset of \( A \times B \):

(a) "\( f : A \to B \) is a function" becomes \( \forall a \in A \exists b \in B \) with \((a, b) \in f \)

(b) "\( f : A \to B \) is injective" becomes \((a, b) \in f \land (a', b') \in f \Rightarrow a = a' \)

(c) "\( f : A \to B \) is surjective" becomes \( \forall b \in B \exists a \in A \) with \((a, b) \in f \)

(d) "Range of \( f \) = \{ b \in B : \exists a \in A \) with \((a, b) \in f \}"

3. Here, edges are unordered pairs of vertices, so our edges are undirected. Since \( \{v, w\} \in E \) must have \( v \neq w \), we have no loops. And because \( E \) is a set, we can have at most one edge between two vertices, so no parallel edges.

:: This represents a simple, undirected graph.

4. In this case, edges are ordered pairs, so our edges are directed. We don't disallow loops \((v, v)\), but because \( E \) is a set, we cannot have parallel edges.

:: This represents a digraph, possibly with loops (but no parallel edges).

5. If the edges are taken from a multiset, we can have parallel edges.
6. If \( G = (V,E) \) and \( G' = (V',E') \) are graphs and \( \phi_v : V \rightarrow V' \), \( \phi_e : E \rightarrow E' \) are bijections:

(a) We need to require that \((e,e') \in \phi_e \Rightarrow (v,v') \in \phi_v \land (w,w') \in \phi_e\), where \( e = (v,w) \) and \( e' = (v',w') \).

(b) The identity functions \( id_v : V \rightarrow V \) and \( id_e : E \rightarrow E \) would show reflexivity; inverses \( \phi_v^{-1} \) & \( \phi_e^{-1} \) would be used to show symmetry; and compositions of bijections would be used to show transitivity.

Also, in each case, the edge/vertex consistency condition of part (a) would need to be checked.

7. Defining \( (x,y) = \{\{x\}, \{x,y\}\} \) gives:

- \( (a,b) = \{\{a\}, \{a,b\}\} \) (if \( a \neq b \))
- \( (b,a) = \{\{b\}, \{a,b\}\} \)

Two sets as elements:
- 4 is the singleton, and
- 6 is the other element of

\( \{a\} \)

\( \{a\} = \{\{a\}\} \)

\( \{a\} = \{\{a\}\} \)

Thus \( (a,b) = (a',b') \) means that either:

* Each has just one set, so \( a=6 \) and \( a'=6' \). Our sets are \( \{a\} = \{a'\} \), so \( a = a' \).

- Not equal: \( a = a' \) and \( b = a = a' \), so \( b = b' \).

- Each has two sets, which must be a singleton and a doubleton. Our sets are \( \{\{a\}\}, \{a,b\} \) and \( \{\{a'\}, \{a',b'\} \), so the singletons match up, i.e. \( a = a' \). But the doubletons also match up, so \( a,b \) = \( a',b' \). Thus \( b = b' \).

In either case \( (a,b) = (a',b') \) if \( a = a' \land b = b' \), which is when ordered pairs are equal.

8. Yes, formally, everything can be formally encoded via sets—this is important as a means of giving our objects well-founded logical meaning, but it would be extremely cumbersome to actually work with them this way, which is why we define key concepts such as functions and graphs and learn to think about them via their properties and what we can prove using those properties, without getting tangled up in more & more complicated sets, sets of sets, sets of sets of sets, etc.