Basic Set Theory & Logic

Swenton

The concepts of set theory and logic are very closely connected, starting with their very basic principles:

- Given a set A and some object x, either $x \in A$ or $x \notin A$, but not both.
- A proposition P is either *true* or *false*, but not both.

In particular, from any set A, we obtain the statement that $x \in A$, connecting sets to statements; under this correspondence, each set concept directly relates to some logical concept, and [almost] vice-versa:

	Set Theory		Logic		Connection
Atomics					
	empty set	$\varnothing$	*false*		$x \in \varnothing \iff$ *false*
	universal set	$*$	*true*		Formally, no "universal set" exists
Unary operation					
	complement	$*$	$\neg$ / !	logical negation	The "complement" of $\varnothing$ would be universal
Binary operations					
	union	$\cup$	$\vee$ / ||	or (inclusive!)	$x \in A \cup B \iff x \in A$ or $x \in B$
	intersection	$\cap$	$\wedge$ / &&	and	$x \in A \cap B \iff x \in A$ and $x \in B$
	set difference	$\setminus$	$\wedge\neg$ / &&!	and-not	$x \in A \setminus B \iff x \in A$ and $x \notin B$
	symmetric diff.	$\triangle$	$\oplus$ / ^	exclusive-or	$x \in A \triangle B \iff x \in A \oplus x \in B$
Relations					
	subset	$\subset$	$\Rightarrow$	implies	$A \subset B$ means $x \in A \Rightarrow x \in B$
	equality	$=$	$\Leftrightarrow$	if and only if	$A = B$ means $x \in A \Leftrightarrow x \in B$
Operations on collections / Logical quantifiers					
	Union	$\bigcup$	$\exists$	existential quantifier	$x \in \bigcup \mathscr{A} \iff \exists A \in \mathscr{A} : x \in A$
	Intersection	$\bigcap$	$\forall$	universal quantifier	$x \in \bigcap \mathscr{A} \iff \forall A \in \mathscr{A}, x \in A$

- On the one hand, these concepts should reinforce one another due to shared meaning (and Venn diagrams!).
- On the other hand, each thing lives in just one world, that of sets or that of logic, so be careful to keep the symbols distinct in your head and your writing!